Light Spanners with Stack and Queue Charging Schemes

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Motivation
Metrical Optimization Problems in Graphs (e.g. TSP)
Previous Work: Charging Schemes
Book Embedding vs. Stack and Queue Charging Scheme
Graph Families for Queue and Stack Schemes

Results for the Talk
Charging Schemes for Bounded Pathwidth Graphs
Outline

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Traveling Salesman Problem

- TSP – NP Complete
- 1-2 TSP – MAX-SNP Hard
- Metric TSP – ∃ A Fast 2 Approximation Algorithm
Metric TSP

- There are approximation algorithms for Metric TSP with bounded errors.
- Have: Error $\leq \epsilon w(G)$
- Want: Error $\leq \epsilon w(MST)$
- Lucky: $w(G') \leq \epsilon w(MST)$ $G'$: pruned graph from $G$
Candidate: *Light Spanners*

$G' = \text{Span}(G, 1 + \epsilon)$ with the following good properties:

1. "Span": for $u, v \in V$, $d_{G'}(u, v) \leq (1 + \epsilon)d_G(u, v)$
2. "Light": $w(G') \leq \frac{k}{\epsilon}w(\text{MST})$
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Charging Scheme

- Charging Scheme (Proved by LP duality)
- For each \((e_i, p_i)\), \(e_i\) pay 1 unit of charge, every \(e \in p_i\) receive 1 unit of charge
- Goal of the Dual Problem: to minimize the value of charges received for edges of trees

\[ p_i = B-C_2-E \]
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Book Embedding vs. Charging Schemes

- **Book Embedding**: A book drawing of $G$ onto a book $B$ should be:
  - every vertex of $G$ is mapped to the spine of $B$; and
  - every edge of $G$ is mapped to a single page of $B$.

- A book embedding of $G$ onto $B$ requires the drawing does not have crossings.
- Every page is (outer)-planar
- Queue Scheme/Queue-compatible Page
- Stack Scheme/Stack-compatible Page
Queue and Stack Charging Schemes

- $(c, d)$-graph
- $c$ – Number of Queue Pages
- $d$ – Number of Stack Pages
- Retrospect: "Light": $w(G') \leq \frac{k}{\epsilon} w(MST)$
- $k = 2c + d$
- If $c, d$ are $O(1) \rightarrow k$ is, too.
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Planar Graphs $\rightarrow (0, 2)$-graphs

- Technique: No Crossing

- Bounded Genus Graphs $\rightarrow (6g - 2, 3g - 2)$-graphs

- Technique: Decompose Bounded Genus Graphs into union of planar graphs
Robertson-Seymour Theory: graphs of minor-closed family can be decomposed into the following components:

1. Bounded Genus Graphs
2. Apices
3. Vortices
4. Clique Sums

Vortices: Bounded Pathwidth Graphs stitched to the surface

Grigni’s conjecture: every minor close graph family has light spanners
To charge Bounded Pathwidth Graphs:

1. Convert it to Bounded Bandwidth Graphs
2. Construct a path by taking an Euler Tour of MST
3. Assume MST is a path, we show a counterexample

\[ \hat{G} \to (O(\sqrt{n}), O(\sqrt{n})) \]-graphs and Bounded Pathwidth
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- Charging Schemes for Bounded Pathwidth Graphs
Convert Bounded Pathwidth Graphs to Bounded Bandwidth Graphs
Bounded Bandwidth Graphs

- Goal: To Bound the **Maximum Degree**
- Assume weight 0 to edges between duplicate vertices
Bounded Pathwidth Graphs: Counterexample

- Solid Line: the MST $T$ of $G'$
- Zig-Zag Line: edges not in $T$ ($e \in G' - T$)
- $O(\sqrt{n})$ Zig-Zag Edges in each group; total $O(\sqrt{n})$ groups
Queue and Stack charging scheme cannot handle bounded pathwidth graphs

However, we are able to solve it by creating a structure called "monotone tree" (http://arxiv.org/abs/1104.4669)

Future Work

- How to connect vortices to the plane or bounded genus graphs?
- How to handle clique sum individually?