New Lower Bounds for Table III

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Abstract

In Radziszowski's Dynamic Survey [1] of Small Ramsey Numbers, Table III gives known bounds for pairs of graphs which are either complete or one edge shy of complete (and at least one of which is not complete). These notes detail constructions that improve several entries in that table.

Introduction

In Radziszowski's Dynamic Survey [1] of Small Ramsey Numbers, Table III gives known bounds for pairs of graphs which are either complete or one edge shy of being complete (at least one of which is not complete). These notes detail constructions that improve several entries. Our new lower bounds are displayed in the table below.

	$K_3 - e$	$K_4 - e$	$K_5 - e$	$K_6 - e$	$K_7 - e$	$K_8 - e$	$K_9 - e$	$K_{10} - e$
$K_3 - e$								
K_3								
$K_4 - e$							34	41
K_4					37			
$K_5 - e$				31	40			
K_5								
$K_6 - e$			31	45				
K_6			37					
$K_7 - e$			40					
K_7			51					

The constructions were made using what might be called the *circulant-block* method, wherein we search for graphs whose adjacency matrices can be decomposed into equal sized square circulant blocks. This method is described in [2].

Constructions of this type can be specified in a rather compact form. We use r to denote the number of blocks in edach direction and b to denote the size of each block. So $n = rb$. Then adjacency matrix A of such a coloring has the following form.

$$A = \begin{pmatrix} B_{1,1} & B_{1,2} & \dots & B_{1,r} \\ B_{2,1} & B_{2,2} & \dots & B_{2,r} \\ \dots & \dots & \dots & \dots \\ B_{r,1} & B_{r,2} & \dots & B_{r,r} \end{pmatrix}$$

Since each block is a circulant, it is sufficient to specify one row from each block. For our colorings, we will specify the elements of the rows that receive color one by using $n(a_1, a_2, \dots a_k)$ to denote the circulant matrix with color 1 entries in positions a_1, a_2, .., a_k and color 2 entries elsewhere. The constructions follow.

$R(K_4, K_7 - e) > 36$:

$$\begin{aligned} B_{1,1} &= 18(3,7,8,9,10,11,15) \\ B_{1,2} &= 18(0,2,5,9,10,11,15) \\ B_{2,1} &= 18(0,3,7,8,9,13,16) \\ B_{2,2} &= 18(1,3,4,6,12,14,15,17) \end{aligned}$$

$R(K_5 - e, K_6) > 36$:

$$\begin{aligned} B_{1,1} &= 18(1,2,6,9,12,16,17) \\ B_{1,2} &= 18(0,2,4,5,9,14,16,17) \\ B_{2,1} &= 18(0,1,2,4,9,13,14,16) \\ B_{2,2} &= 18(3,7,8,9,10,11,15) \end{aligned}$$

$R(K_5 - e, K_6 - e) > 30$:

$$\begin{aligned} B_{1,1} &= 10(3,7) \\ B_{1,2} &= 10(2,4,5,7) \\ B_{1,3} &= 10(0,3,6,7,8,9) \\ B_{2,1} &= 10(3,5,6,8) \\ B_{2,2} &= 10(1,3,7,9) \end{aligned}$$

$$\begin{aligned}
B_{2,3} &= 10(0,1,6,7)\\
B_{3,1} &= 10(0,1,2,3,4,7)\\
B_{3,2} &= 10(0,3,4,9)\\
B_{3,3} &= 10(1,9)
\end{aligned}$$

$R(K_5 - e, K_7) > 50$:

$$\begin{aligned}
B_{1,1} &= 10(3,4,6,7)\\
B_{1,2} &= 10(1,2,3,7,8)\\
B_{1,3} &= 10(6,8,9)\\
B_{1,4} &= 10(2,4,5,6,8)\\
B_{1,5} &= 10(4,9)\\
B_{2,1} &= 10(2,3,7,8,9)\\
B_{2,2} &= 10(3,4,6,7)\\
B_{2,3} &= 10(1,4,6)\\
B_{2,4} &= 10(3,8)\\
B_{2,5} &= 10(1,2,3,5,9)\\
B_{3,1} &= 10(1,2,4)\\
B_{3,2} &= 10(4,6,9)\\
B_{3,3} &= 10(1,3,7,9)\\
B_{3,4} &= 10(1,2,4,5,8)\\
B_{3,5} &= 10(0,5,9)\\
B_{4,1} &= 10(2,4,5,6,8)\\
B_{4,2} &= 10(2,7)\\
B_{4,3} &= 10(2,5,6,8,9)\\
B_{4,4} &= 10(1,9)\\
B_{4,5} &= 10(1,2,4,6,7,9)\\
B_{5,1} &= 10(1,6)\\
B_{5,2} &= 10(1,5,7,8,9)\\
B_{5,3} &= 10(0,1,5)\\
B_{5,4} &= 10(1,3,4,6,8,9)\\
B_{5,5} &= 10(1,2,8,9)
\end{aligned}$$

$R(K_5 - e, K_7 - e) > 39$:

$$\begin{aligned}
B_{1,1} &= 13(1,3,5,8,10,12)\\
B_{1,2} &= 13(0,1,8,9,11)\\
B_{1,3} &= 13(3,4,6,8,9)\\
B_{2,1} &= 13(0,2,4,5,12)\\
B_{2,2} &= 13(2,3,10,11)\\
B_{2,3} &= 13(0,1,2,3,7,9,12)\\
B_{3,1} &= 13(4,5,7,9,10)\\
B_{3,2} &= 13(0,1,4,6,10,11,12)\\
B_{3,3} &= 13(2,3,10,11)
\end{aligned}$$

$R(K_6 - e) > 44$:

$$\begin{aligned}
B_{1,1} &= 22(1,3,7,8,9,10,11,12,13,14,15,19,21)\\
B_{1,2} &= 22(1,4,5,6,10,11,13,18,21)\\
B_{2,1} &= 22(1,4,9,11,12,16,17,18,21)\\
B_{2,2} &= 22(1,3,7,8,9,10,11,12,13,14,15,19,21)
\end{aligned}$$

$R(K_4 - e, K_{10} - e) > 40$:

$$\begin{aligned}
B_{1,1} &= 10(5)\\
B_{1,2} &= 10(1,5,8,9)\\
B_{1,3} &= 10(5,6,7,8)\\
B_{1,4} &= 10(6)\\
B_{2,1} &= 10(1,2,5,9)\\
B_{2,2} &= 10(1,5,9)\\
B_{2,3} &= 10(2,5)\\
B_{2,4} &= 10(2,6,9)\\
B_{3,1} &= 10(2,3,4,5)\\
B_{3,2} &= 10(5,8)\\
B_{3,3} &= 10(4,6)\\
B_{3,4} &= 10(3,5,6,8)\\
B_{4,1} &= 10(4)\\
B_{4,2} &= 10(1,4,8)
\end{aligned}$$

$$
\begin{aligned}
B_{4,3} &= 10(2,4,5,7) \\
B_{4,4} &= 10(1,4,6,9)
\end{aligned}
$$

For the case of $R(K_4 - e, K_9 - e)$ the graph has $n = 33$, $r = 11$ and $b = 3$, so it is probably best to simply give the adjacency matrix.

$R(K_4 - e, K_9) > 33$:

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022 211 221 222 212 212 122 221 222 222 111
202 121 122 222 221 221 212 122 222 222 111
220 112 212 222 122 122 221 212 222 222 111

211 022 221 222 212 212 122 221 111 222 222
121 202 122 222 221 221 212 122 111 222 222
112 220 212 222 122 122 221 212 111 222 222

212 212 011 221 122 212 222 221 221 221 122
221 221 101 122 212 221 222 122 122 122 212
122 122 110 212 221 122 222 212 212 212 221

222 222 212 022 221 111 112 222 221 221 122
222 222 221 202 122 111 211 222 122 122 212
222 222 122 220 212 111 121 222 212 212 221

221 221 122 212 011 221 122 122 212 212 221
122 122 212 221 101 122 212 212 221 221 122
212 212 221 122 110 212 221 221 122 122 212

221 221 221 111 212 022 222 121 222 222 222
122 122 122 111 221 202 222 112 222 222 222
212 212 212 111 122 220 222 211 222 222 222

122 122 222 121 122 222 022 222 122 121 212
212 212 222 112 212 222 202 222 212 112 221
221 221 222 211 221 222 220 222 221 211 122

212 212 212 222 122 112 222 022 222 111 222
221 221 221 222 212 211 222 202 222 111 222
122 122 122 222 221 121 222 220 222 111 222

222 111 212 212 221 222 122 222 022 212 121
222 111 221 221 122 222 212 222 202 221 112
222 111 122 122 212 222 221 222 220 122 211

222 222 212 212 221 222 112 111 221 022 122
222 222 221 221 122 222 211 111 122 202 212
222 222 122 122 212 222 121 111 212 220 221

111 222 122 122 212 222 221 222 112 122 022
111 222 212 212 221 222 122 222 211 212 202
111 222 221 221 122 222 212 222 121 221 220
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The adjacency matrices for these colorings, as well as some other material, can be obtained from the Web site

`http://isu.indstate.edu/ge/RAMSEY.`

References

[1] S. P. Radziszowski. *Small Ramsey Numbers.* Dynamic Survey DS1, Electronic J. Combinatorics. 1 (1994-1999).

[2] G. Exoo, *Some New Ramsey Colorings*, Electronic J. Combinatorics. 5 (1998), #R29.