Extremely Large Prime Numbers with Repeated Digits

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Results So Far

- Test the number in the format of $6 \times 10^k + 1$.
- The largest prime I get:
- $6 \times 10^{20812}+1$.
- 600000000000000000000000000000000…..1
- 20811 zeros in this number

Prime Number Theorem

- π (x) ~ $\frac{\boldsymbol{x}}{\boldsymbol{ln(x)}}$ =the number of prime numbers that below x.
- For example: $\pi(20)$=8, there are 8 prime numbers below 20 (2,3,5,7,11,13,17,19). $\pi(100)$=25, since there are 25 prime numbers below 100.
- So the possibility to pick a random number is prime is $\frac{\mathbf{1}}{\boldsymbol{ln(x)}}$.
- Application: can be used to make a guess about how long it will take to find primes of different sizes.

How Many Numbers Need To Be Tested

- If we pick numbers of d digits, there are $10^d - 10^{d-1}$ numbers total. Using the prime number theorem, there would be $\pi(10^d) - \pi(10^{d-1})$ prime numbers.

- So the chance to pick a prime number is:
$$\frac{\pi(10^d) - \pi(10^{d-1})}{10^d - 10^{d-1}} \approx \frac{1}{ln(d)}$$

The Time To Check One Number

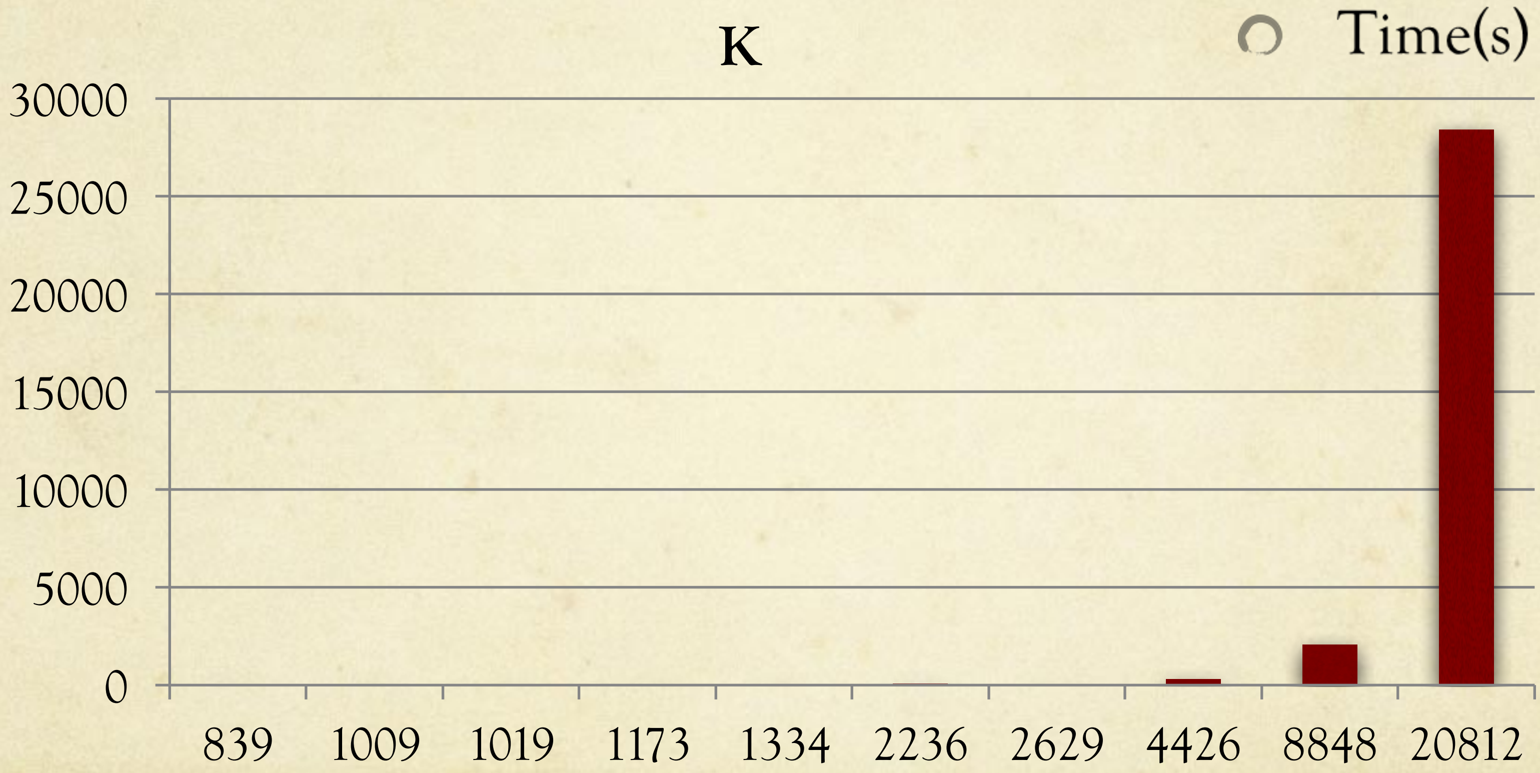
- Sometimes we are lucky and it is easy to tell that a number is not prime (like it if it is even or a multiple of 3).
- Otherwise, the program runs a Fermat primality test, which takes roughly ($\#digits^2$) time.

Time To Find Prime

- From the prime number theorem we get the idea that it may take a very long time to find a large prime.
- Combined the chance of finding a prime and the time to check one number, we have a hypothesis that the running time is in the format c*d^3 (time for checking each number * the math expectation of finding prime)when d is large enough.

Running Time

- Running time: the time for the computer to find the prime



K	Time (seconds)	Time (hours)
839	17	--
1009	11	--
1019	1	--
1173	11	--
1334	11	--
2236	77	--
2629	41	--
4426	302	0.08
8848	2074	0.58
20812	28412	7.89

Comparison

Number:	EXPECTED TIME(c*d^3)	Real time	Deviation:
$6*10^{1173}+1$	1.52	11	9.48 (86%)
$6*10^{1334}+1$	15.97	11	4.97 (45%)
$6*10^{2236}+1$	51.79	77	25.21 (32%)
$6*10^{2629}+1$	123.93	41	82.93 (202%)
$6*10^{4426}+1$	194.81	302	107.19 (35%)
$6*10^{8848}+1$	2416	2074	342 (16.48%)
$6*10^{20812}+1$	26940	28412	1472 (5%)

Comparison Conclusion

- When K is small, the running time seems randomly.
- When K is really large, the real time and the expected time are very close.
- Time to find a prime with 100,000 digits: 3145360.75 sec, 873 hours, 36 days.
- Time to find a prime with 1000,000 digits: 3145360753.41 sec, 873711 hours, 36406 days, 101 years.

Future Research

- Run the program on many computers to make the test faster.