

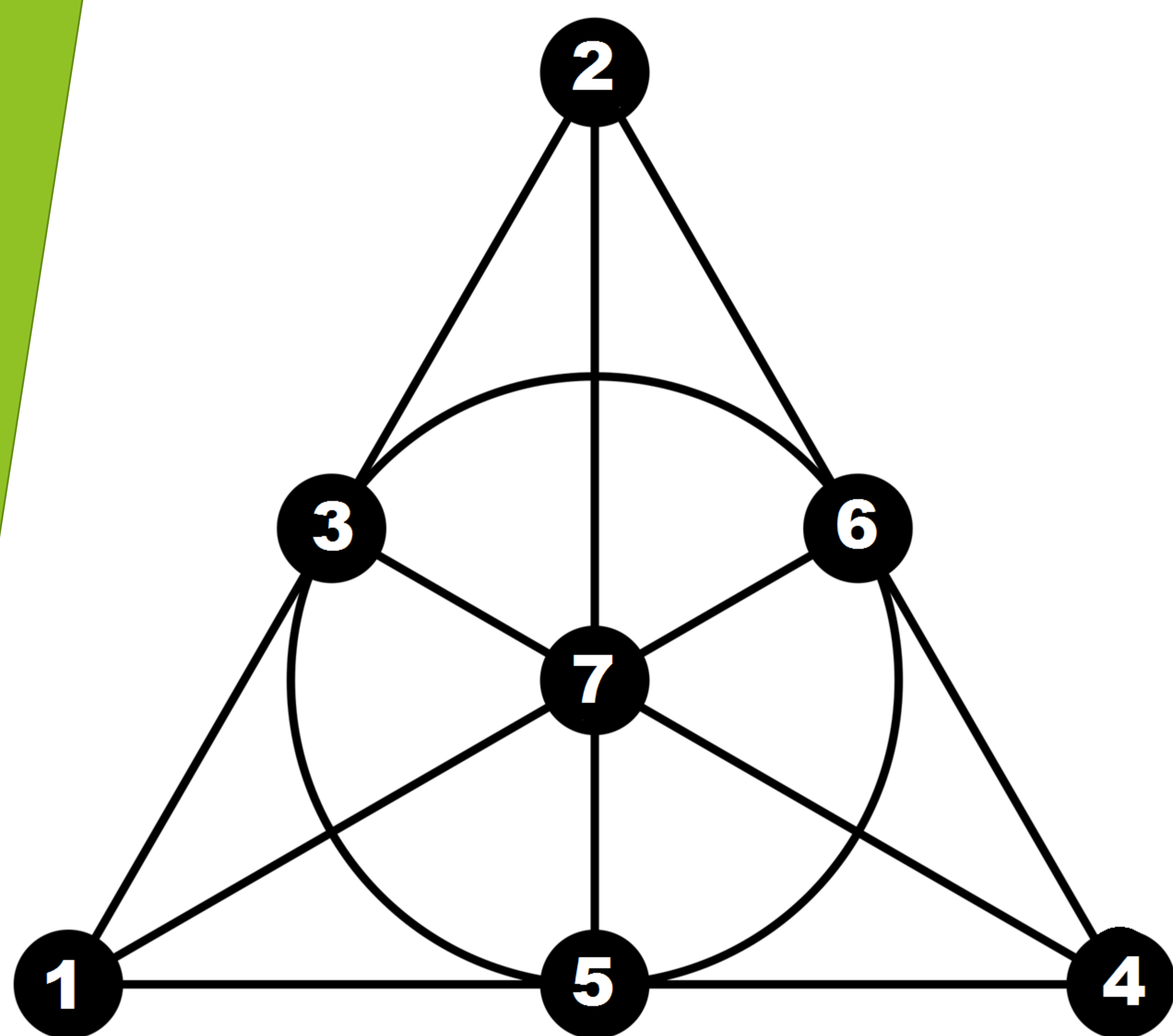
Estimating the Number of Steiner Triple Systems of Order 21

Jacob Sonntag

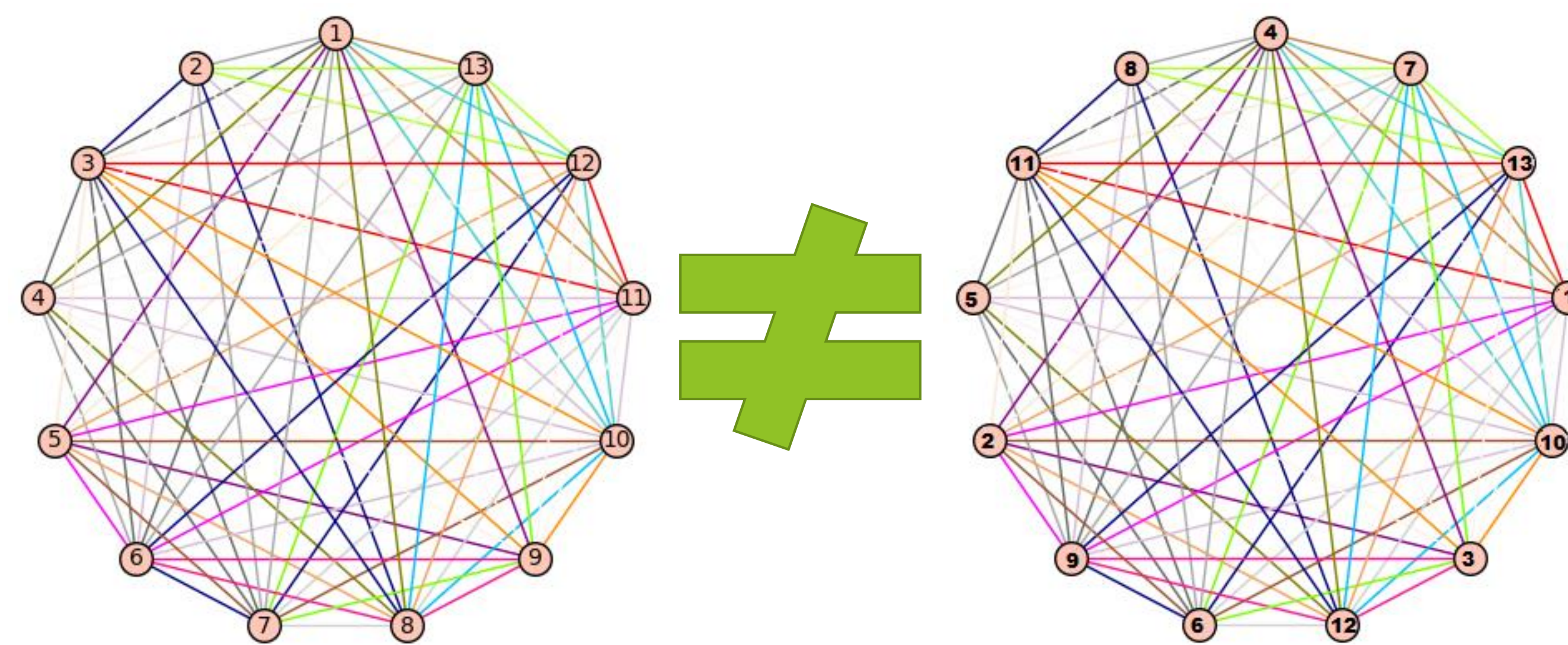
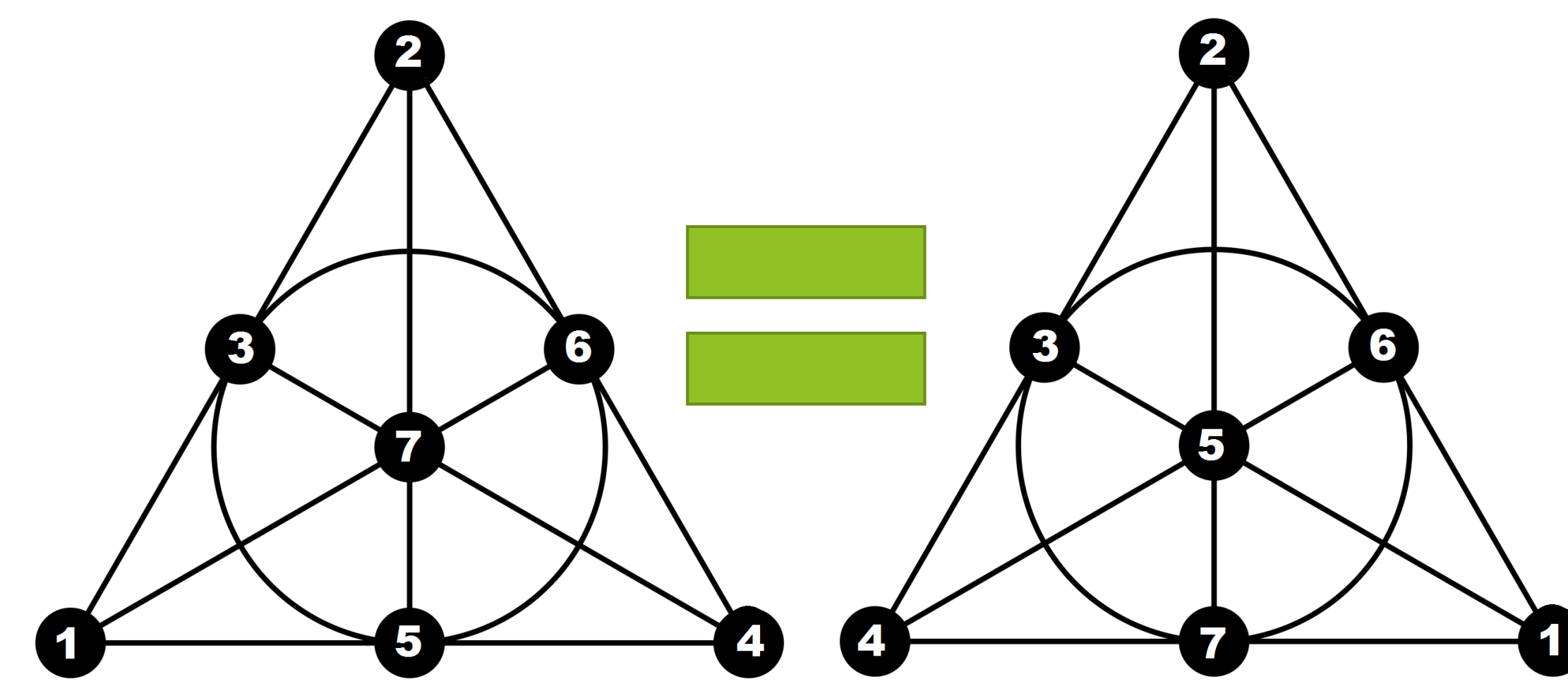
Steiner Triple System

Make a group of elements. A Steiner Triple System takes those elements and splits them up into blocks, where each block has 3 elements from the original group (each element can be in multiple blocks). Every pair of elements belongs to a unique block. This only works when the initial group has 7, 9, 13, 15... elements.

$$n \equiv 1, 3 \pmod{6}$$



Steiner Triple System of Order 7
(Fano Plane)



Isomorphisms

Two STSs are 'isomorphic' if there exists a permutation from the elements of the former to the latter. If there is no such permutation, then they are not isomorphic.

Estimating by Sampling with Replacement

An analogy: Imagine you have a bag filled with marbles, where each marble represents a STS of a certain order. If two STSs are isomorphic, then their corresponding marbles have the same color. By picking out marbles, noting their color, then replacing them into the bag, you can get a general idea of how many colors are in the bag, and therefore how many isomorphisms of that order exist.

Number of Isomorphisms of Steiner Triple Systems of Order n

7	9	13	15	19	21
1	1	2	80	11,084,874,829	???