

CS 621, Assignment 2

Deadline: October 20 (5:00 pm)

Problem 0.1 Find a solution for a_n in the following recurrence relations.

- $a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 3 + 5n$, $a_0 = 1$, $a_1 = 2$, $a_2 = 3$.
- $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$, $a_0 = 1$, $a_1 = 2$.

Problem 0.2 Let $1 \leq k \leq n$ be two integers. Show that :

1. $\binom{n}{k} \leq n^k / 2^{k-1}$.
2. $\binom{2n}{n} \geq 4^n / 2n$.

Problem 0.3 (15 points) Solve the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2} + 5(2^{2n+1})$, $n \geq 2$ with $a_0 = 1$ and $a_1 = 1$.

Problem 0.4 We apply a sequence of pushes and pops (of length $2n$) on sequence $1, 2, 3, \dots, n$ (coming to a Stack on this order). If we apply a push step then a new number is going to be added on the top of the Stack. If we apply pop operation a number comes out of the top of the stack and we print it out.

For example if the input sequence is $1, 2, 3$ and we apply the "push,push,pop,push,pop,pop" operations on it we will get a sequence $2, 3, 1$.

The output sequence is called a Stacki sequence. What is the number of Stacki sequences of length n ? Find a recurrence relation.

Problem 0.5 Let $\Sigma = \{0, 1\}$ and $A = \{0, 01, 011, 0111, 1111\} \subset \Sigma$. Let a_n count the number of string in A^* of length n . Find a_n (use recurrence relation).

Problem 0.6 What is the number of n -digit quaternary $(0, 1, 2, 3)$ sequences in which there is never a 3 anywhere to the right of 0 ?