Course Information and Introduction

Arash Rafiey

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Course Information

1. **Instructor : Arash Rafiey**
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   - Office : Root Hall A-127
   - Office Hours : Tuesdays 12:00 pm to 1:00 pm in my office (A-127)
2. **Course Webpage :** http://cs.indstate.edu/~arash/ads.htm

Course Information

Objective : Cover the topics : inclusion-exclusion, generating functions, recurrence relations, graph theory

1. Advanced Probability : (Review of Finite Probability, Conditional Probability)
2. The Principle of Inclusion-Exclusion (review) Generalized Inclusion-Exclusion
3. Advanced Enumeration : Introduction to Generating Functions, Calculation Techniques, Partitions of Integers
4. Recurrence Relations : The Method of Generating Functions
5. Automorphism groups of combinatorial structures,
6. Number Theory and Crypthography
7. error correcting codes
8. Graph Theory : Subgraphs, Complements, and Graph Isomorphism Vertex Degree: Euler Trails and Circuits Planar Graphs, Hamilton Paths and Cycles Graph Coloring and Chromatic Number

Course Information and Grading

Textbooks :

Discrete Combinatorial Mathematics: An Applied Introduction Edition: (5th edition), Ralph P. Grimaldi, Pearson Education 2004.

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Grading :

1. Homework assignments (4 assignments each 5 %)
2. Midterm (25 %)
3. Final (55 %)

Counting

Show that $24 | m(m+1)(m+2)(m+3)$.

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What is the number of ways of choosing m elements from an n-elements set ?

Pigeon hole principal

Show that in a party of 6 people there are three people such that :

- they are pairwise friends, or
- no pair of them are friends

Background checking

Show that in every tournament there is a directed path going through all the nodes.

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If G is a *d-tree* (maximum degree is d) then prove that G has at least $\lfloor \frac{|V|}{d} \rfloor$ cut vertices.

Background checking

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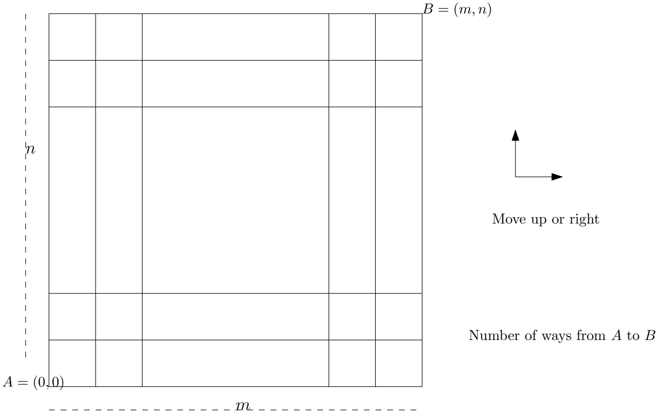
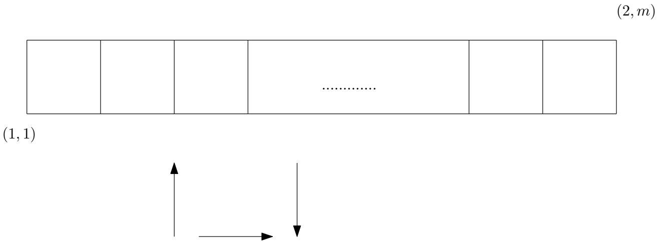


Background checking

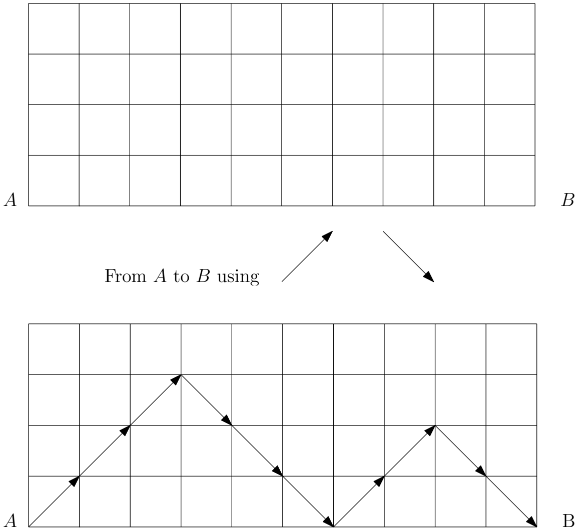
In an $2 \times m$ grids we want to move from $(1, 1)$ to $(2, m)$ using the following rules :

- moving one step up or one step right or one step down.
- if we move down then we can not move up immediately.
- if we move up then we can not move down immediately.



Background checking

Going from A to B using one unit diagonal moves $\nearrow$, $\searrow$.



Definition : We say a sequence S of $0, 1$ is **nice** if the number of ones and the number of zeros are the same and

in every prefix of S the number of ones is not less than the number of zero.

Problem : What is the number of nice strings of length $2n$?

Background checking

How we compute the number of nice sequences ?

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Let C_n be the number of nice-sequences of length $2n$.

Consider the first index i that the number of 1's and the number of 0's (from 1 to $2i$) are the same.

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$C_0 = 1,\ C_1 = 1.$

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$C_n = \frac{1}{n+1}\binom{2n}{n}$

Background checking

This problem is similar to the number of ways of multiplying n-matrixes :

$$(A_1(A_2(A_3A_4)))$$
$$(A_1((A_2A_3)A_4))$$
$$((A_1A_2)(A_3A_4))$$
$$((A_1(A_2A_3))A_4)$$
$$(((A_1A_2)A_3)A_4)$$