Trees

Arash Rafiey

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Proof.

Since T is connected, there is a path from a to b. If there is another path from a to b then we get a cycle by union the two paths. □

Theorem

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Proof.

We use induction on the number of edges. If $|E| = 0$ then it is clear that T has only one vertex. Let $e = uv$ be an edge in T. By removing e we would have two subtrees $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$. Observe that $|E_1| < |E|$ and $|E_2| < |E|$. By induction hypothesis $|T_1| = |E_1| + 1$ and $|T_2| = |E_2| + 1$. We have $|T| = |T_1| + |T_2|$ and $|E| = |E_1| + |E_2| + 1$. Therefore $|T| = |E| + 1$.

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Proof.

Set $n = |T|$. We know that $|E| = n - 1$. If there are k leaves then $2(n-1) = 2|E| = \sum_{v \in V} deg(v) \geq k + 2(n-k)$. Therefore $k \geq 2$. □

Labeling Trees

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From each labeling we obtain a sequence as follows.

1) Start with $i = 1$.

2) Set x_i to be the parent of the smallest labeled leaf y_j.

3) Remove the leaf and increase i by one. If $i = n - 2$ stops otherwise go to (2).

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If G is a directed graph, then G is called a *directed tree* if the undirected graph associated with G is a tree. When G is a directed tree, G is called rooted tree if there is a unique vertex r, called *root* , in G with indegree zero, and for all other vertices v, the in degree of v is 1.

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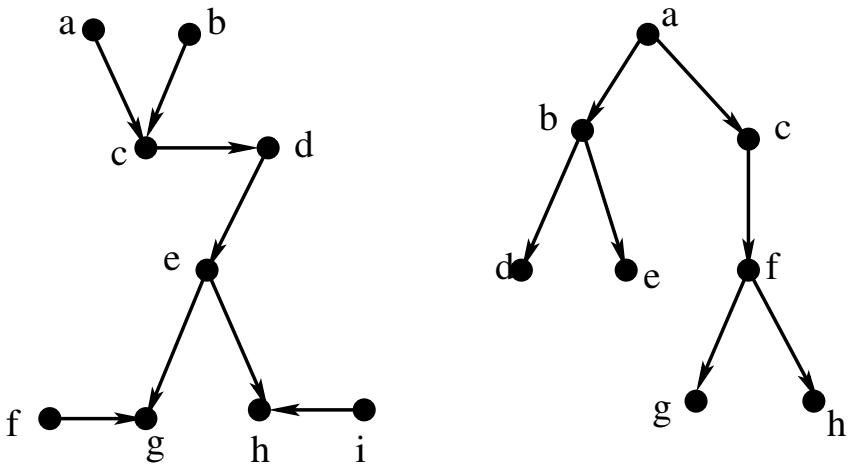

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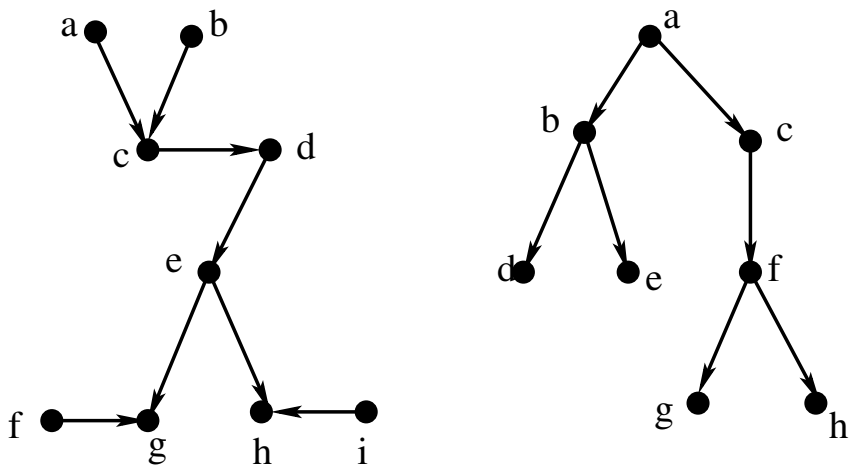

For simplicity we may ignore the direction once the root identified.

Traversing a tree starting from root

Breadth-First Search Method : Visiting the vertices of the tree level by level starting from root r

Breadth-First Search (BFS). Start with node r and set $L_0 = r$.

At step i let L_i be the set of nodes that are not in any of

$L_0, L_1, \ldots, L_{i-1}$ and have a neighbor in L_{i-1}.

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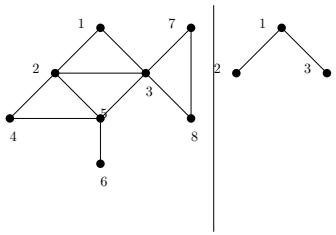

Depth-First Search (backtracking approach)

We don't visit the nodes level by level! As long as there is an unvisited node adjacent to the current visited node we continue! Once we are stuck, trace back and go to a different branch!

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PreorderDFS (r)

1. Mark r as Explored and print r.
2. For every edge rv
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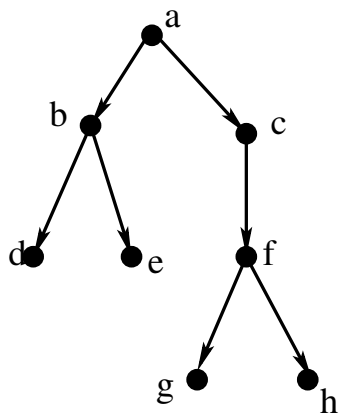

Pre–DFS : a, b, d, e, c, f, g, h

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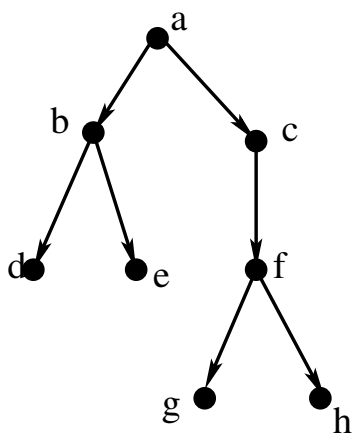

Post–DFS : d, e, b, g, h, f, c, a