Hard Problems (NP problems)

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There has been a lot of works trying to classify hard problems and there has been a lot of progress in this regard.

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We can verify a given answer to problem X is valid or not by spending a polynomial amount of time $\mathcal{O}(|X|^c)$, c is constant number (independent from $|X|$).

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If $|I| < k$ then we say no.

Otherwise :
We consider every pair of nodes u, v in I and if there is an edge between them then we say no. Otherwise we say yes.

It takes $\mathcal{O}(|I|^2)$ and since $|I| \leq n$, it takes $O(n^2)$ to verify.

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Check whether the sequence is a Hamiltonian cycle
Check whether its cost is at most W.

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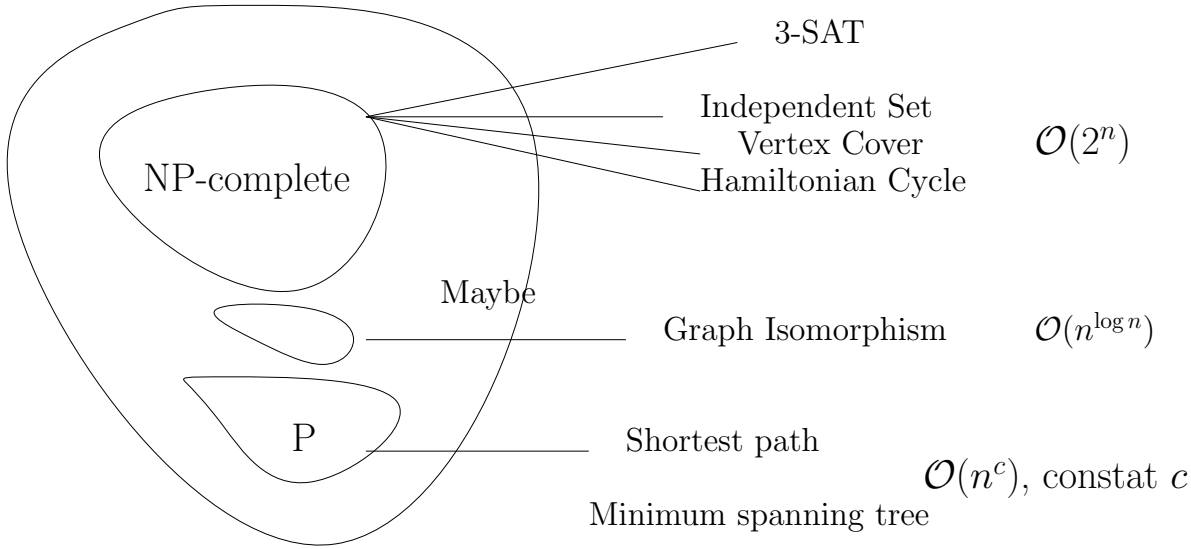
For $x = y = z = 0, v = w = 1$ F is not satisfiable.

For $x = y = 0, z = v = w = 1$ F is satisfiable.

If there are m clauses and n variables in F then we can check in $\mathcal{O}(mn)$ whether F is satisfiable or not (the size of F is the number of clauses + the number of variables in F).

NP is the class of problems X where verifying the solution for an instance $x \in X$ takes $\mathcal{O}(|x|^c)$, c is a constant .



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Example : Circular Scheduling (independent set when we have circular arcs) Problem Y.
We use Interval Scheduling solution (procedure, tool box) to solve an instance of Circular Scheduling (call n times).

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Suppose $Y \leq_p X$. If Y can not be solved in polynomial time, then X can not be solved in polynomial time.

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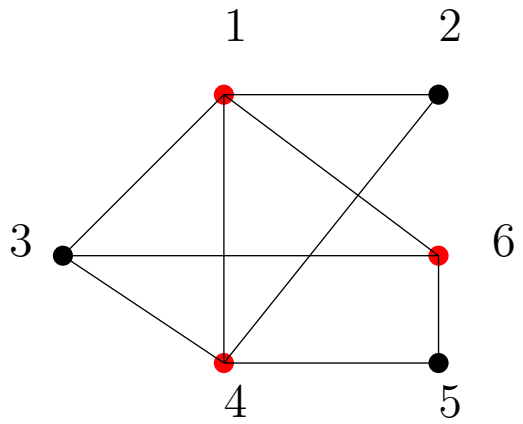

$S = \{1, 4, 6\}$

Problem

Given a graph G and a number k, does G contain a vertex cover of size at most k ?

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Conversely, suppose $V - S$ is a vertex cover for G. Now for every edges uv of E, at least one of the u, v is in $V - S$ and hence S is an independent set. □

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More General Case Reduction : Vertex Cover to Set Cover

Set Cover : Given a set U of n elements, a collection $S_1, S_2, \ldots, S_m$ of subsets of U, and a number k, does there exists a collection of at most k of these sets whose union is equal to all of U.

$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

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S_i for $i \in V$ be the set of edges $\{ij_1, ij_2, \ldots, ij_t\}$; ($j_1, j_2, \ldots, j_t$ are the neighbors of i in G).

Proof.

Now if we can cover U by picking at most k sets then G has a vertex cover of size at most k.

If $S_{i_1}, S_{i_2}, \ldots, S_{i_\ell}$, $\ell \leq k$ are the sets to cover U, then every edge in E is incident to one of $i_1, i_2, \ldots, i_\ell$. So $i_1, i_2, \ldots, i_\ell$ is a vertex cover.

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Conversely, if $\{i_1, i_2, \ldots, i_\ell\}$ is a vertex cover of size $\ell \leq k$, then the sets $S_{i_1}, S_{i_2}, \ldots, S_{i_\ell}$ is a cover for U.

Reduction from 3SAT

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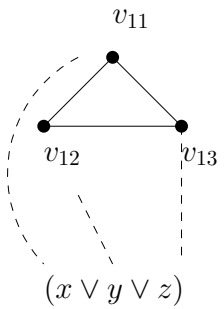
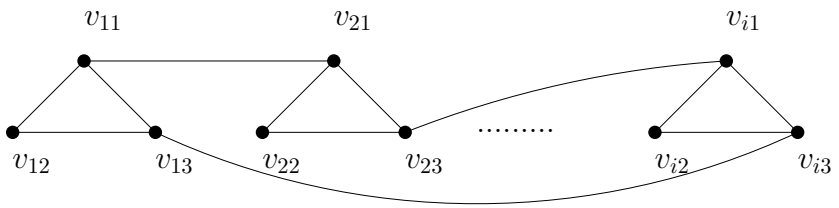
3-SAT $\leq_p$ Independent Set. We want to show that Independent Set is at least as hard as 3- Satisfiability .

We should transform an arbitrary instance of 3-SAT to an instance of an independent set.
Let $F = C_1 \wedge C_2 \wedge \cdots \wedge C_k$.
For each clause $C_i = (x \vee y \vee z)$ we consider three nodes v_{i1}, v_{i2}, v_{i3} representing x, y, z and make a triangle.



Our intuition is if we choose v_{i1} in the independent set then we set $x = 1$.

We also note that if we set $x = 1$ then $\neg x = 0$ and the node representing $\neg x$ should not be in the independent set. Therefore : We connect v_{i1} to v_{j1} if $C_j = (\neg x \vee u \vee w)$ or connect v_{i1} to v_{j2} if $C_j = (u \vee \neg x \vee w)$.



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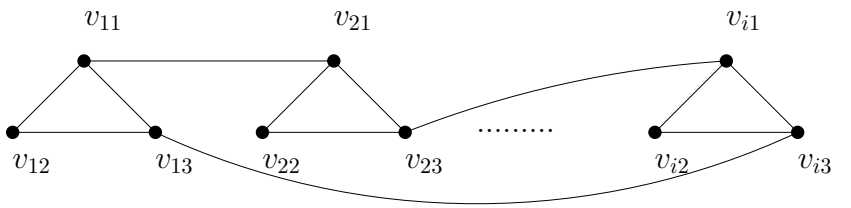

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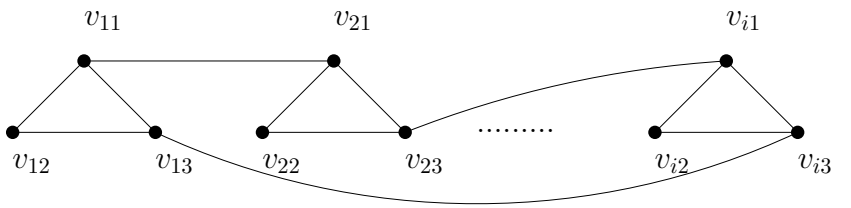

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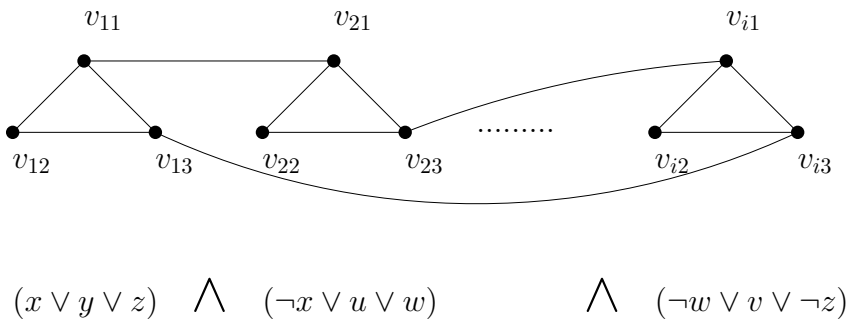

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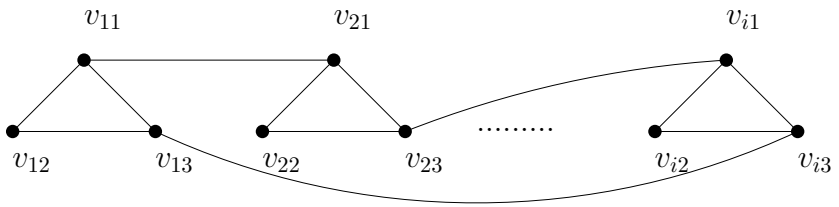
From each of the triangles we choose exactly one node in the independent set. Therefore for each clause at least one literal is set to 1 and hence F is satisfied.



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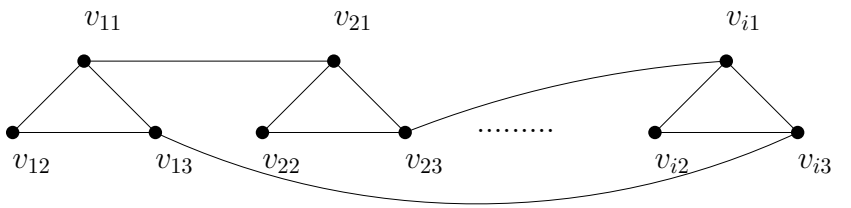

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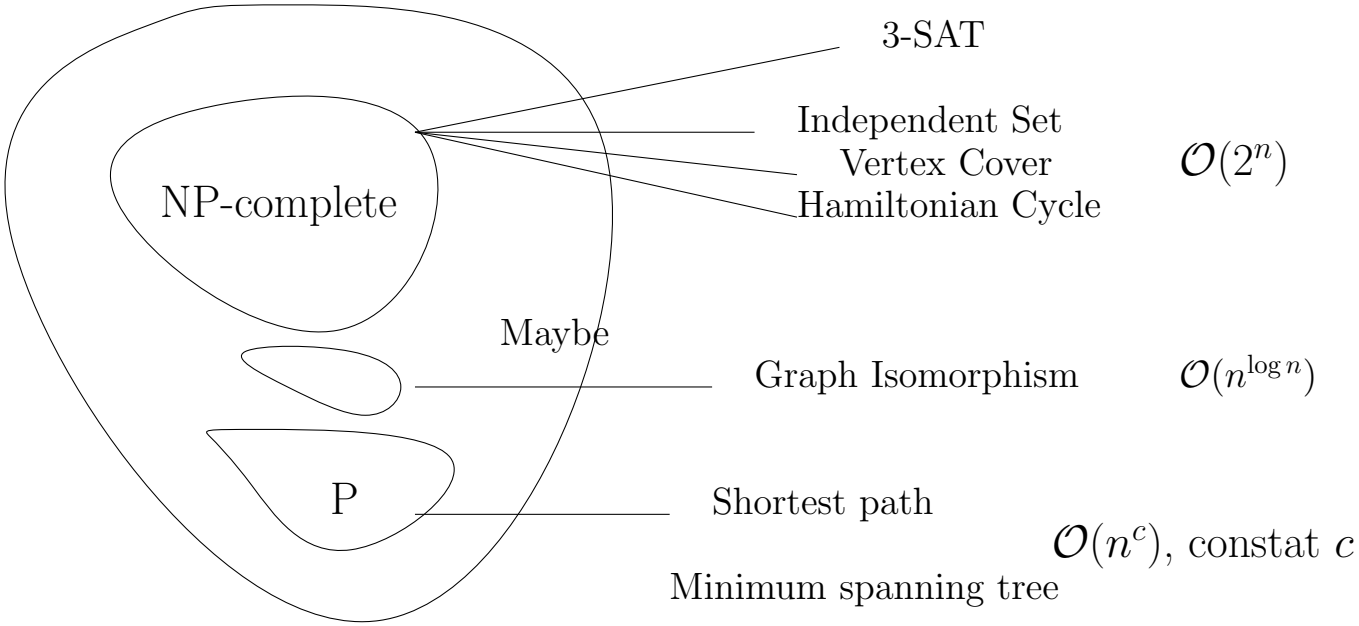
Conversely : We also observe that if F is satisfied then there is an independent set in G of size at least k (if x in clause C_i is 1 then add x to the independent set, and don't add anything else from clause C_i to independent set)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Independent Set
Vertex Cover
Hamiltonian Cycle
$\mathcal{O}(2^n)$
NP-complete
Maybe
Graph Isomorphism
$\mathcal{O}(n^{\log n})$
P
Shortest path
$\mathcal{O}(n^c)$, constat c
Minimum spanning tree

Hitting Set Problem:

Definition

We are given a set $U = \{e_1, e_2, e_3, \cdots, e_n\}$ and $S_1, S_2, \cdots, S_m$ are subsets of U and integer k. The goal is to find a subset of U with size at most k which has intersection with all $S_1, S_2, \cdots, S_m$.

Theorem

Hitting Set Problem is NP complete.

Proof.

We reduce Vertex Cover to Hitting Set!
From an arbitrary instance of vertex cover we construct an instance of hitting set.

□

How??

NP completeness of Hitting Set Problem:

Proof.

1) Consider graph $G = (V, E)$;
2) $U = V(G)$;
3) $S_i = \{u, v\}$; where (u, v) is an edge of G.
4) If C is a vertex cover for G of size $k \Rightarrow$ by definition then, for every edge (u, v) in G either $u \in C$ or $v \in C$. C is solution for Hitting Set because it must intersect every set S_i
5) Now suppose H is a hitting set of U of size k. Since H intersects every set, it has at least one endpoint of every edge. Set C to be H.

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Dominating set problem is NP-complete

We are give a graph $G = (V, E)$ and an integer k. Decide whether there exists a set $S \subseteq V$ such that :

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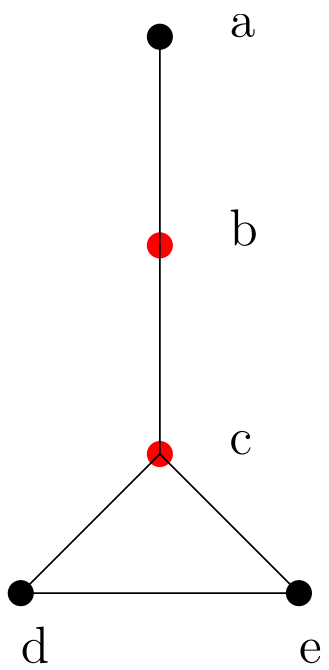

b,c is a dominating set

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For each set S_i we consider a white node and for each element of the ground set U we consider a black node.

We connect a black node v to white node S_i if $v \in S_i$.

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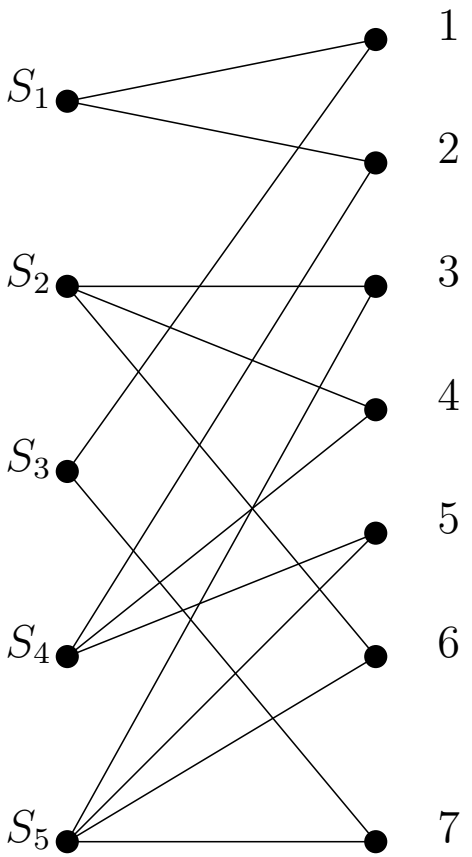

S_2, S_3, S_4 is a set cover since

$S_2 \cup S_3 \cup S_4 = \{1, 2, 3, 4, 5, 6, 7\}$

$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

$$S_1 = \{1, 2\} \qquad S_2 = \{3, 4, 6\} \quad S_3 = \{1, 7\}$$

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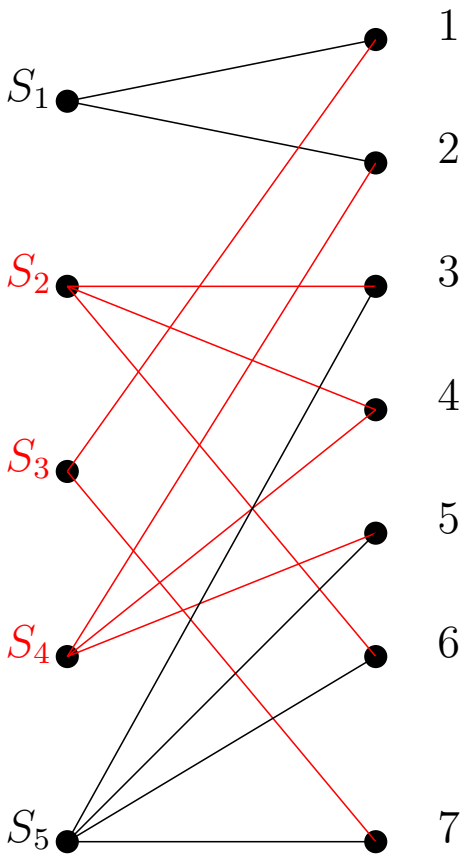

S_2, S_3, S_4 is a set cover since

$S_2 \cup S_3 \cup S_4 = \{1, 2, 3, 4, 5, 6, 7\}$

S_2, S_3, S_4 dominates all right nodes

Now we make a clique on the white vertices (on the set nodes).

$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

$$S_1 = \{1, 2\} \qquad S_2 = \{3, 4, 6\} \quad S_3 = \{1, 7\}$$

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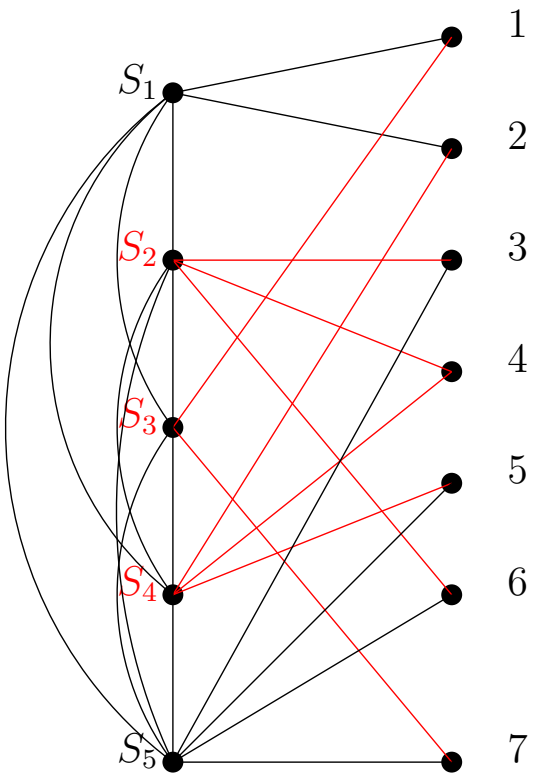

S_2, S_3, S_4 is a set cover since

$S_2 \cup S_3 \cup S_4 = \{1, 2, 3, 4, 5, 6, 7\}$

S_2, S_3, S_4 dominates all right nodes

S_2, S_3, S_4 domiates all others

Important : We should show that if there is a k collections of sets that covers entire U then there is set of nodes in new graph G that is dominating set.

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Let $S_{i_1}, S_{i_2}, \ldots, S_{i_k}$ be a set cover for U.

Then set S of white nodes corresponding to $S_{i_1}, S_{i_2}, \ldots, S_{i_k}$ is a dominating set.

Because every black node has a neighbor in S and sice every white node is adjacent to all other black nodes, S is a dominating set.

Conversely : From a dominating set of size k we find a solution for set cover.

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So we can assume that DS contains only white vertices.