Two Big Questions: P vs. NP and P vs. BPP

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Indiana State University, March 26, 2010

Computational Complexity Theory

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  - My work: derandomization, hierarchy theorems

Introducing two Big Questions

Time Complexity

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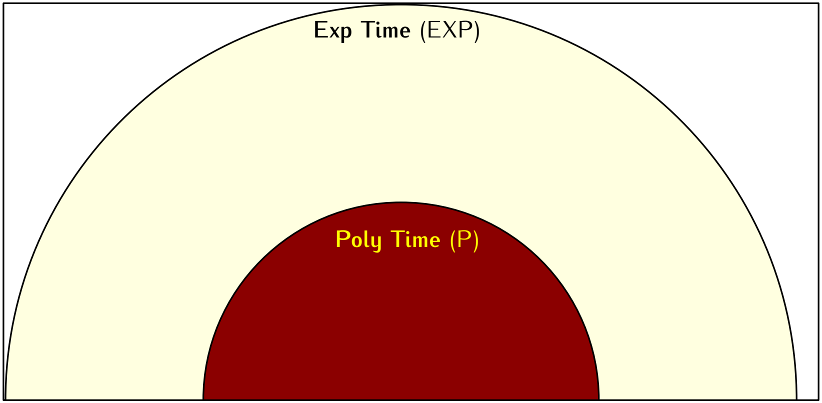

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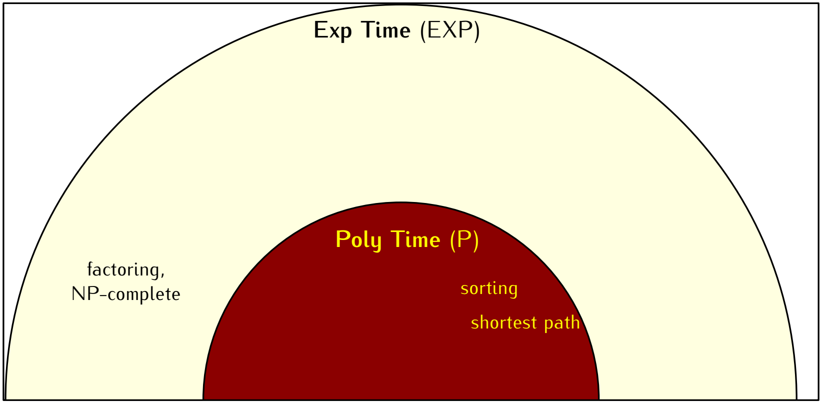

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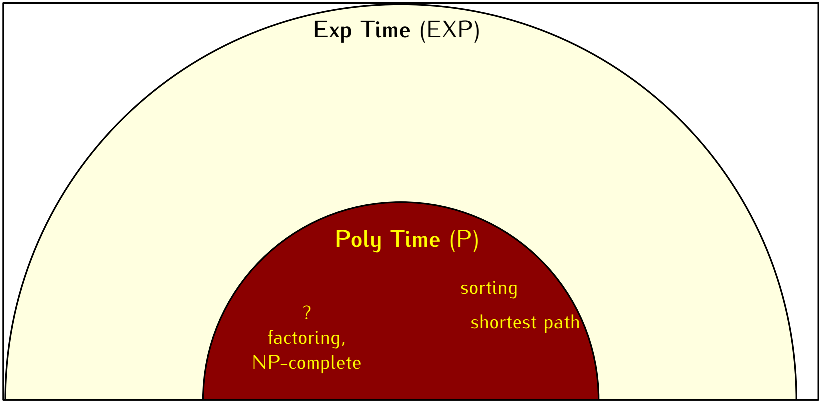

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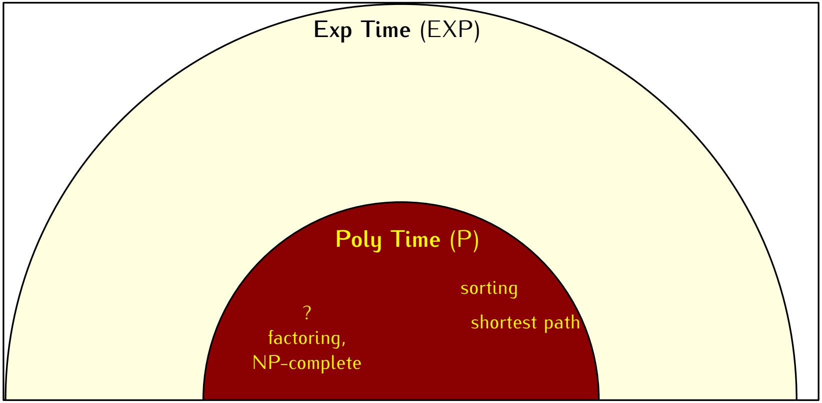

Decision Problem: yes/no questions

Memory Space Complexity

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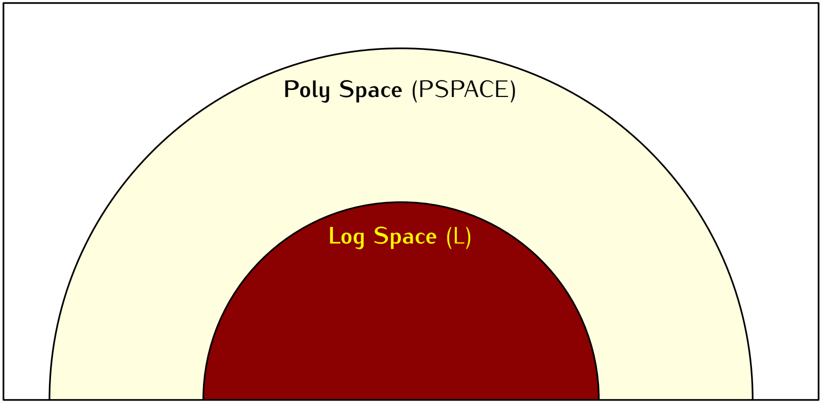

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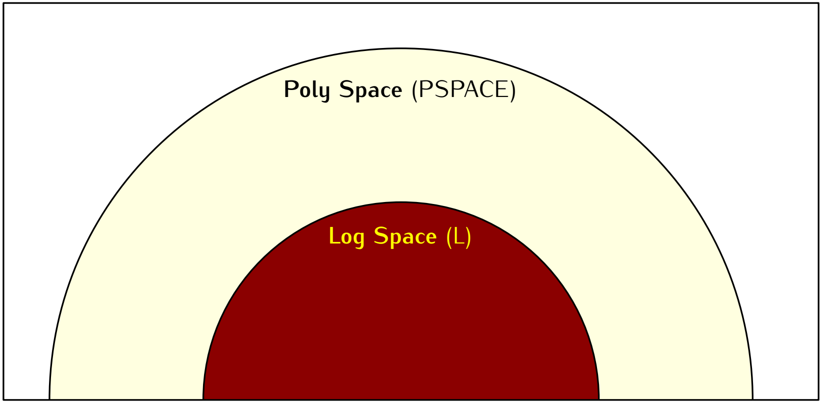
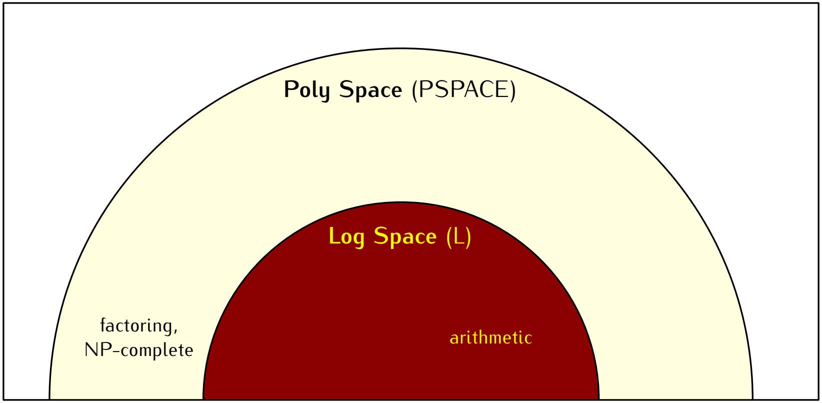


Amount of working space memory needed

Memory Space Complexity



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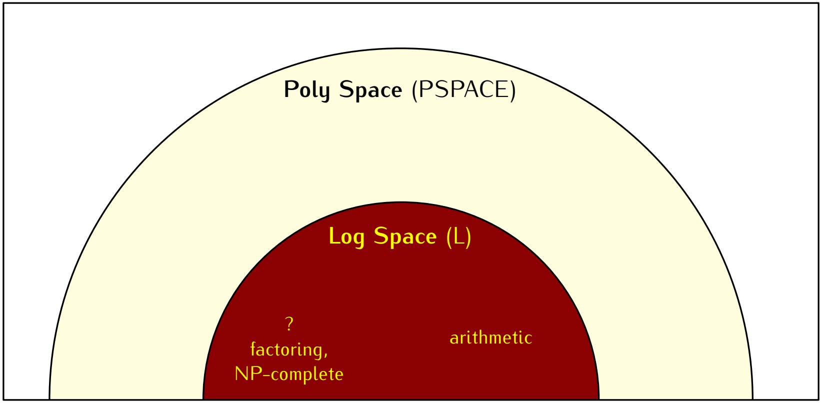

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Randomized Algorithm

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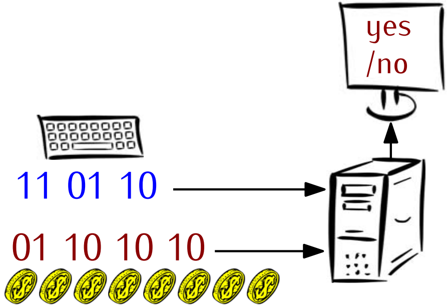

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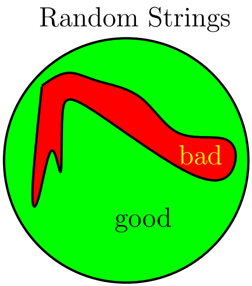

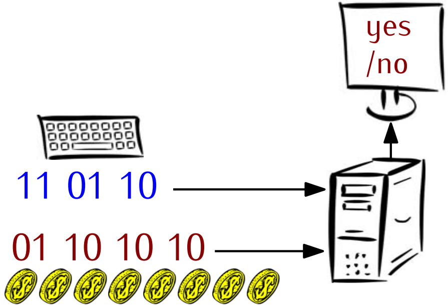

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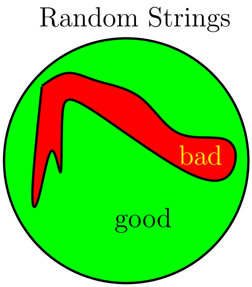

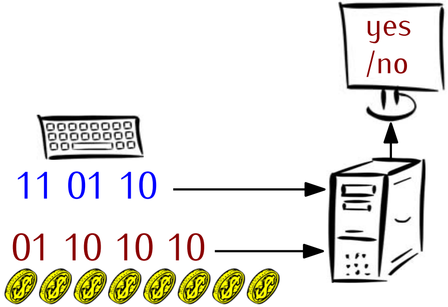

Bounded error: Correct with probability $> 99\%$

Polynomial Identity Testing

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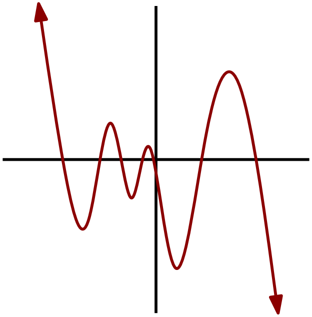

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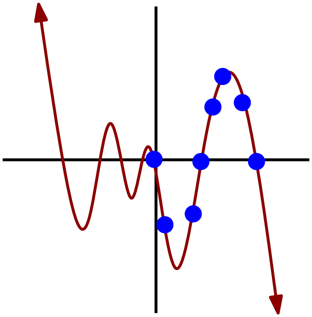

Multi-variate Polynomial Identity Testing

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- $p(x_1, x_2, x_3, x_4) = x_4^5 \cdot (x_1 - x_2 - x_3)^{30} + (x_4 + x_3 - x_1)^{15} \cdot (x_3 - x_2 + x_1)^{20} - (x_2 - x_3 + x_4)^{20} \cdot (x_2 + x_1)^{15}$

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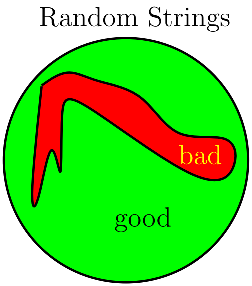
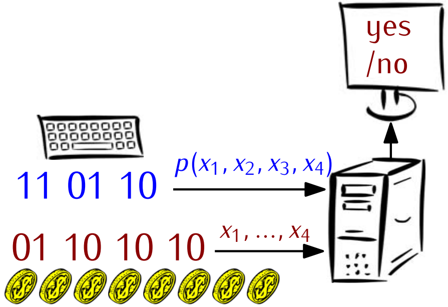
- p non-zero, degree $d \Rightarrow \Pr[p(x_1, ..., x_4) = 0] \leq \frac{d}{|S|}$

Randomized Algorithm





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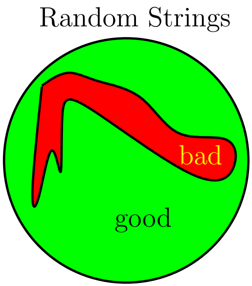

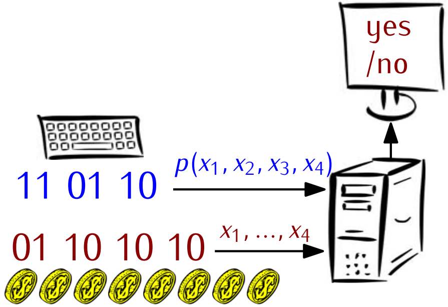

- **BPP**: Bounded-error Probabilistic Poly time

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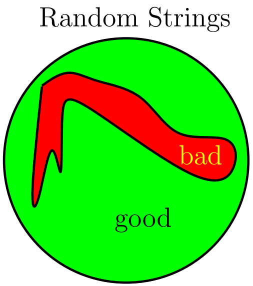

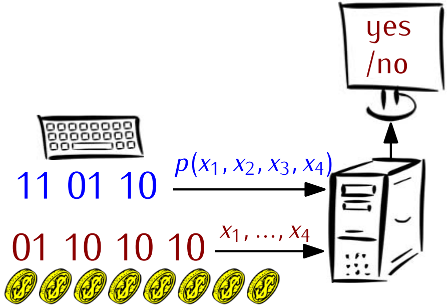

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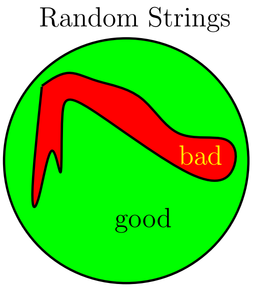

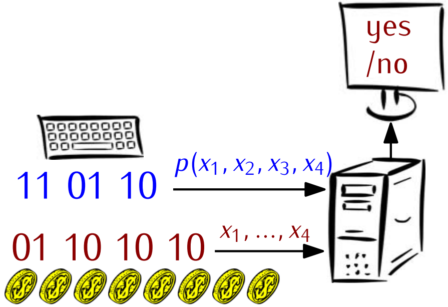

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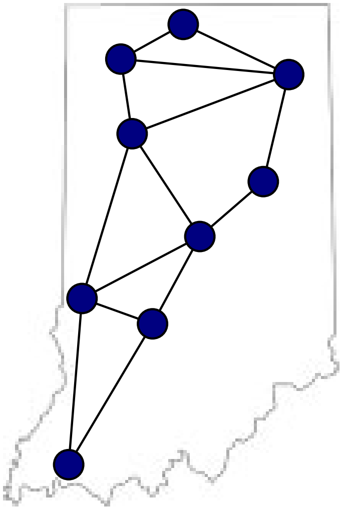
- **P $\overset{?}{=}$ BPP**

Nondeterministic Algorithm

Graph 3-Coloring



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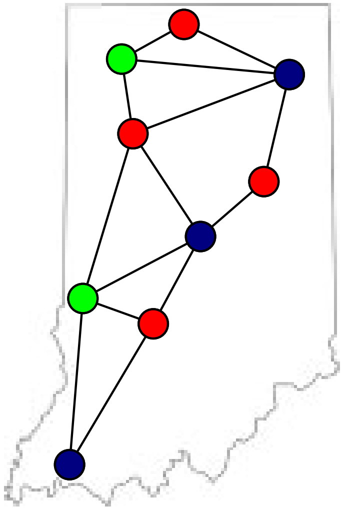

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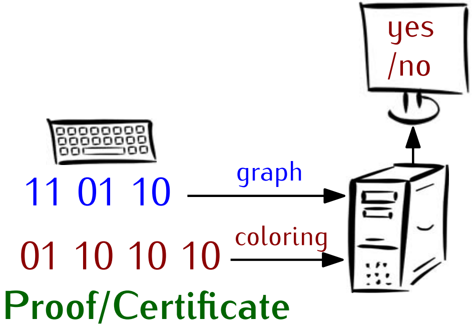

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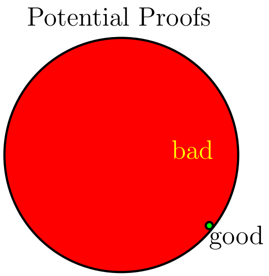

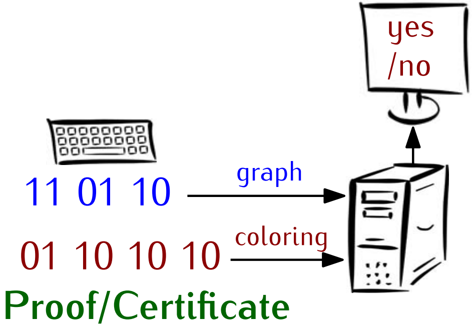

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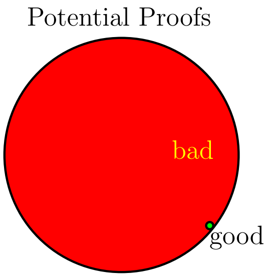

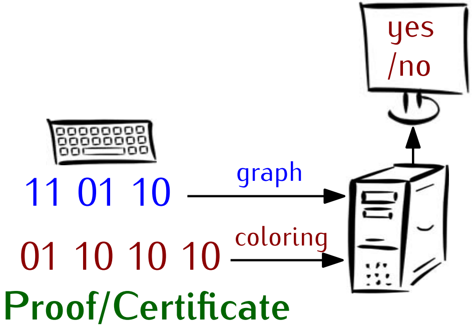

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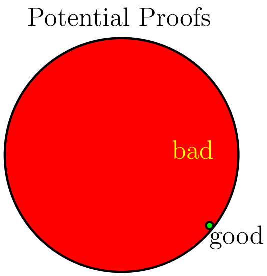

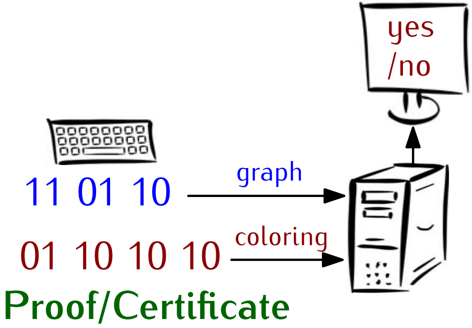

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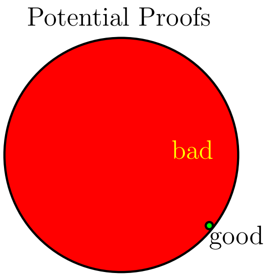

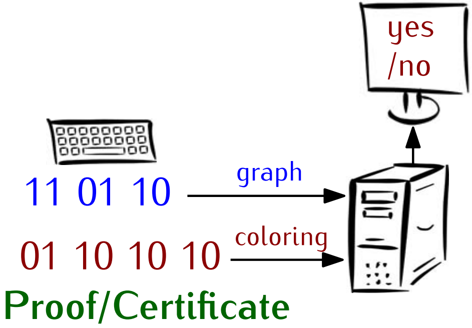

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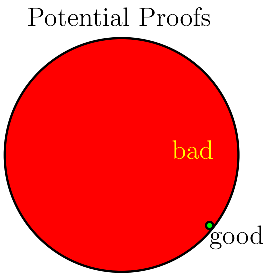

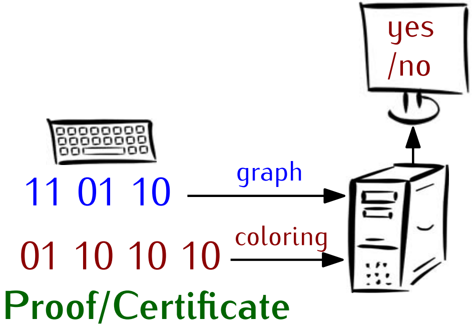

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Two Big Questions

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Is finding proofs as easy as verifying them?

Is 3-coloring in Polynomial Time?

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$$\mathbf{P} \overset{?}{=} \mathbf{BPP}$$

Does randomness $\underline{\text{truly}}$ add power?

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Derandomization

Naive Derandomization

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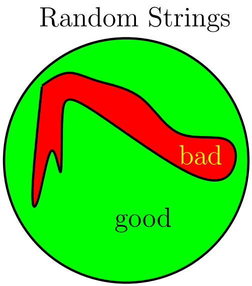
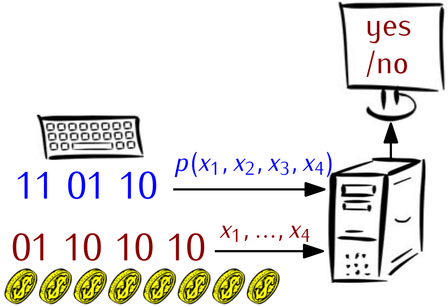
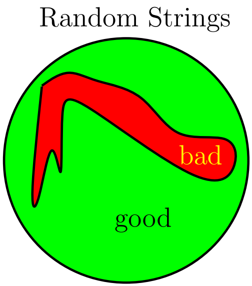
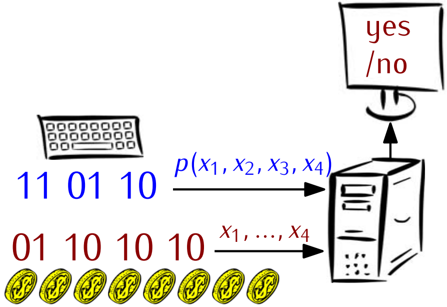




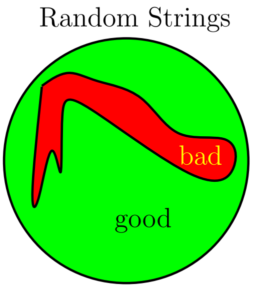
Naive Derandomization

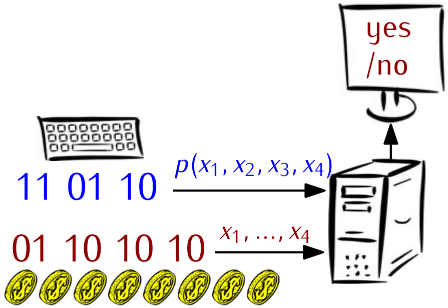




Try all possible random bit strings

Naive Derandomization





Try all possible random bit strings – exponentially many

Derandomization – the Standard PRG Approach

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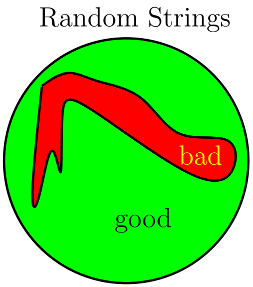

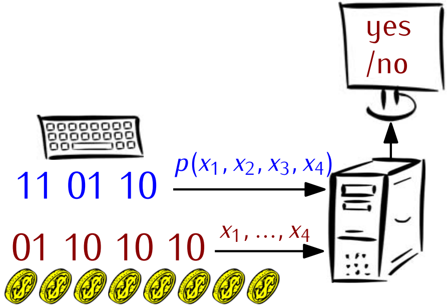

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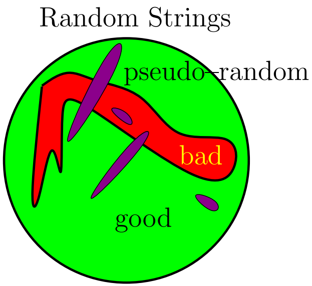

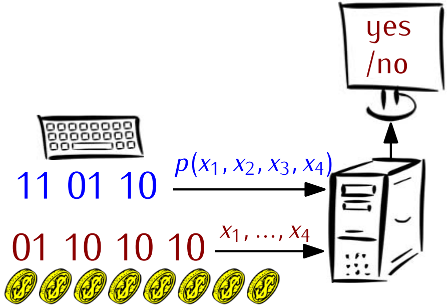

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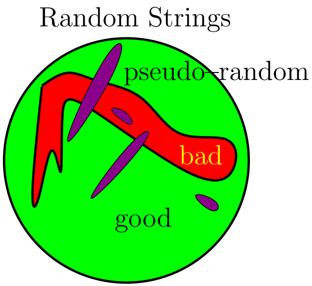

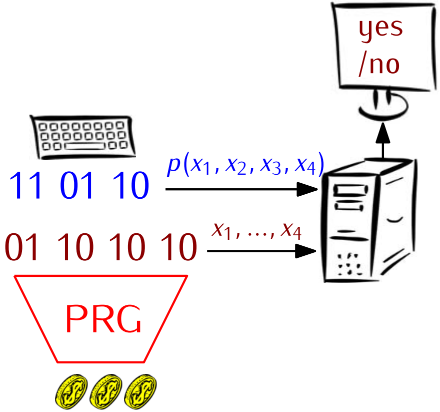

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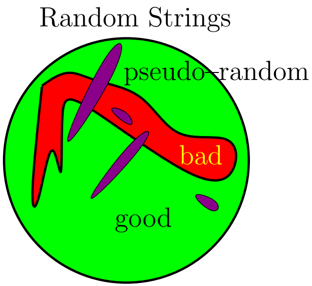

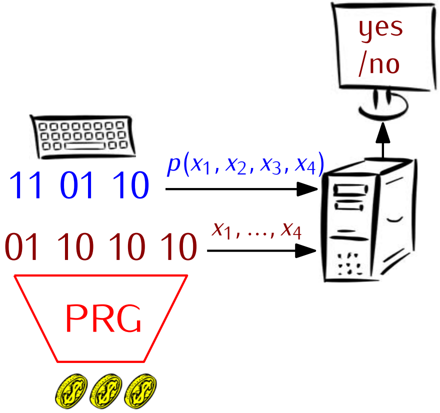

Poly many strings to try

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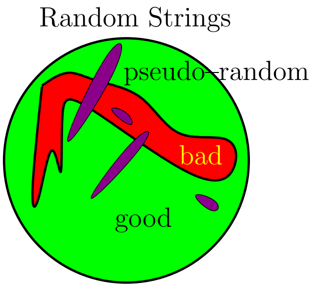
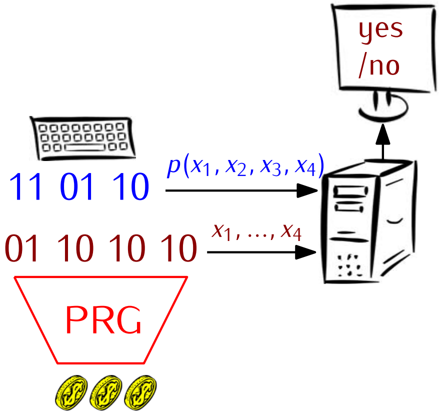
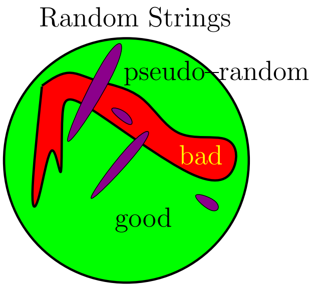
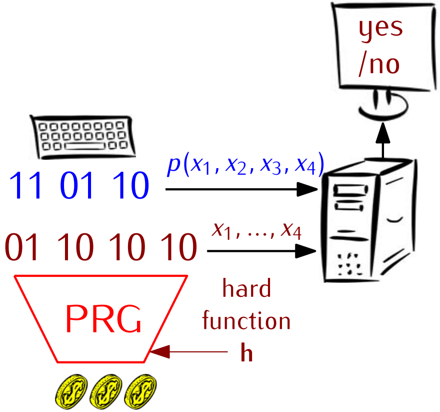




Poly many strings to try $\Rightarrow$ $O(\log n)$ seed, exp stretch

Derandomization – the Standard PRG Approach

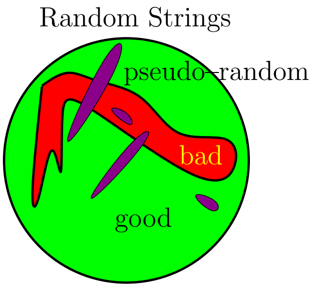


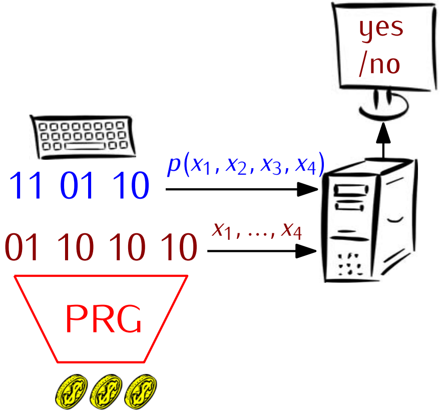


Poly many strings to try $\Rightarrow$ $O(\log n)$ seed, exp stretch

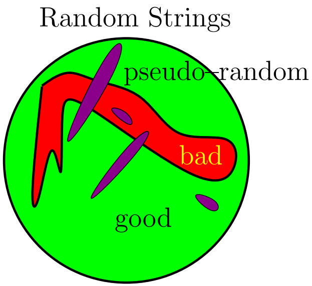
A New Approach – Typically-Correct Derandomization

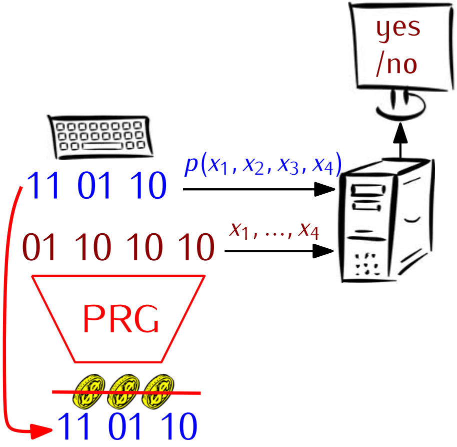
A New Approach – Typically-Correct Derandomization



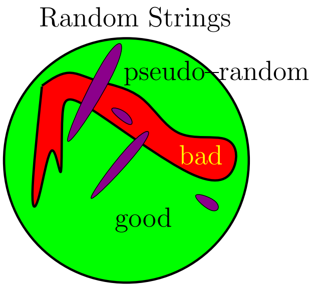


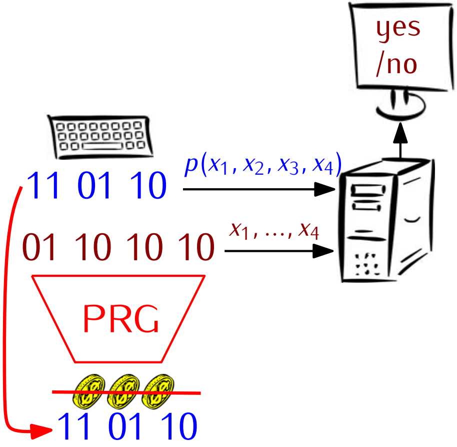
A New Approach – Typically-Correct Derandomization





A New Approach – Typically-Correct Derandomization





Seed length n, poly stretch

Standard Use of PRG's vs. Typ-Correct

Standard Use of PRG's vs. Typ-Correct

Standard Derandomization	**Typ-Correct Derandomization**
[Nisan & Wigderson, ...]	[Kinne, Van Melkebeek, Shaltiel]

Standard Use of PRG's vs. Typ-Correct

Standard Derandomization [Nisan & Wigderson, ...]	**Typ-Correct Derandomization** [Kinne, Van Melkebeek, Shaltiel]
• Always correct	• Small # mistakes

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Standard Derandomization [Nisan & Wigderson, ...]	**Typ-Correct Derandomization** [Kinne, Van Melkebeek, Shaltiel]
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• Always correct	• Small # mistakes
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Standard Use of PRG's vs. Typ-Correct

Standard Derandomization [Nisan & Wigderson, ...]	**Typ-Correct Derandomization** [Kinne, Van Melkebeek, Shaltiel]
• Always correct	• Small # mistakes
• Run PRG many times	• Run PRG only once
• Need exponential stretch	• Need only poly stretch
• Conditional results	• Unconditional results: fast parallel time, streaming, communication protocols

Computational Complexity Theory

How much **time, memory space, etc.** are needed to solve problems?

- **Is nondeterminism powerful?** P vs. NP
  - Conjecture: P$\neq$ NP
  - Techniques: hierarchy theorems, others
- **Is randomness powerful?** P vs. BPP, L vs. BPL
  - Conjecture: P=BPP, L=BPL
  - My work: derandomization, hierarchy theorems

Hierarchy Theorems

Hierarchy Theorems

Hierarchy Theorems

- Fix a model of computing

Hierarchy Theorems

- Fix a model of computing (deterministic, randomized, nondeterministic)

Hierarchy Theorems

- Fix a model of computing
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- Can we achieve more given more resources?

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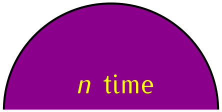

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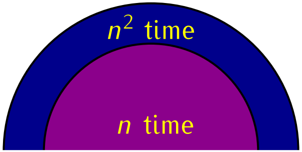

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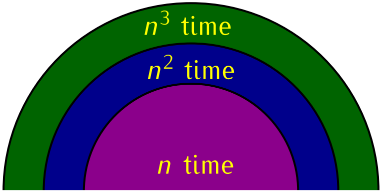

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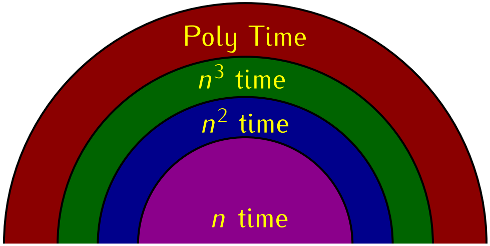

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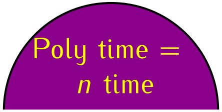

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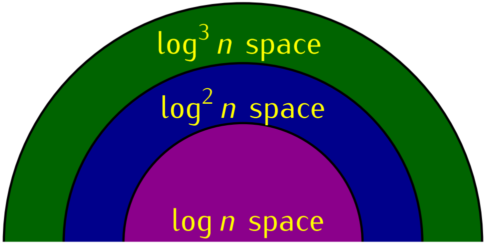

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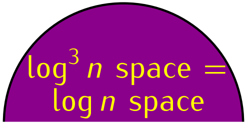

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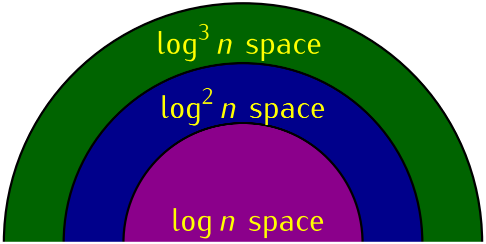

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- My work: hierarchy theorems for randomized algorithms

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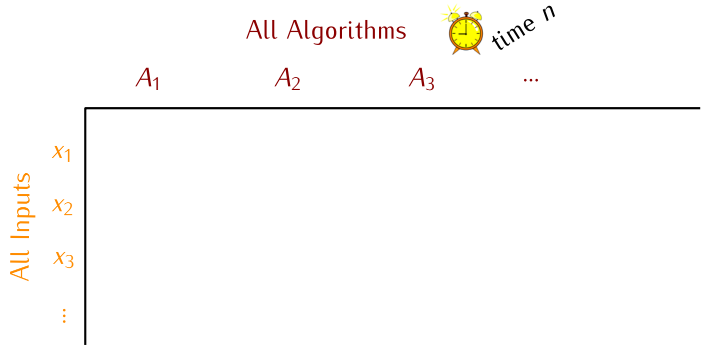
Hierarchy Theorems for Deterministic Algorithms

Hierarchy Theorems for Deterministic Algorithms

All Algorithms time n

A_1 A_2 A_3 ...

Hierarchy Theorems for Deterministic Algorithms



Hierarchy Theorems for Deterministic Algorithms

All Algorithms time n

All Inputs

$$\begin{array}{c|cccc}
 & A_1 & A_2 & A_3 & \cdots \\
\hline
x_1 & A_1(x_1) & A_2(x_1) & A_3(x_1) & \\
x_2 & A_1(x_2) & A_2(x_2) & A_3(x_2) & \cdots \\
x_3 & A_1(x_3) & A_2(x_3) & A_3(x_3) & \\
\vdots & & \vdots & &
\end{array}$$

Hierarchy Theorems for Deterministic Algorithms

All Algorithms

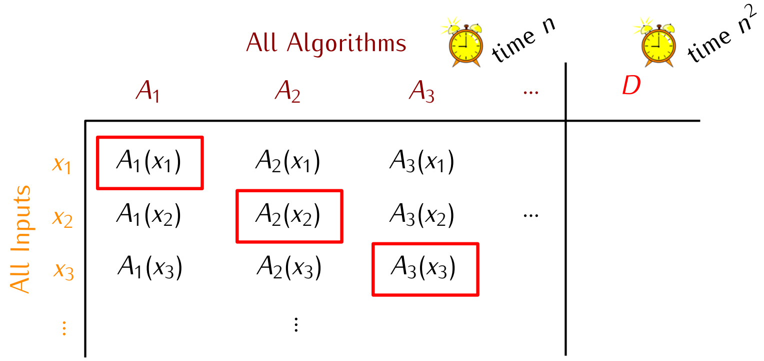
time n

time n^2

All Inputs

	A_1	A_2	A_3	$\cdots$	D
x_1	$A_1(x_1)$	$A_2(x_1)$	$A_3(x_1)$		
x_2	$A_1(x_2)$	$A_2(x_2)$	$A_3(x_2)$	$\cdots$	
x_3	$A_1(x_3)$	$A_2(x_3)$	$A_3(x_3)$		
$\vdots$		$\vdots$			

Hierarchy Theorems for Deterministic Algorithms

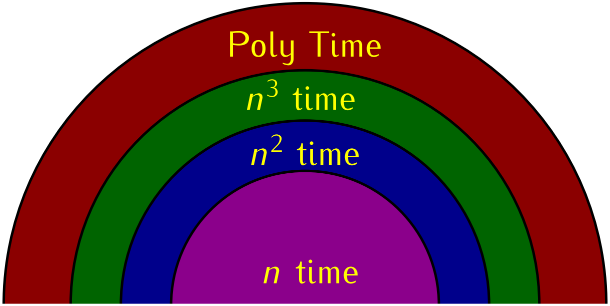


Hierarchy Theorems for Deterministic Algorithms

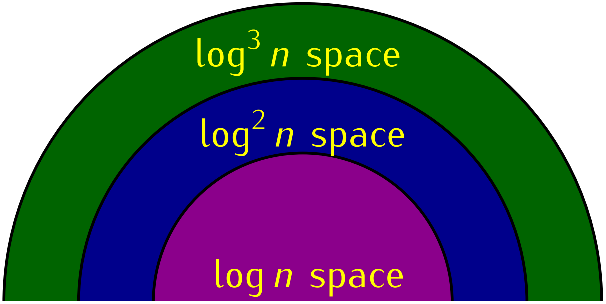
All Algorithms time n time n^2

$$\begin{array}{cc|cccc|c}
 & & A_1 & A_2 & A_3 & \dots & D \\
\hline
 & x_1 & \boxed{A_1(x_1)} & A_2(x_1) & A_3(x_1) & & \neg A_1(x_1) \\
\text{All Inputs} & x_2 & A_1(x_2) & \boxed{A_2(x_2)} & A_3(x_2) & \dots & \neg A_2(x_2) \\
 & x_3 & A_1(x_3) & A_2(x_3) & \boxed{A_3(x_3)} & & \neg A_3(x_3) \\
 & \vdots & & \vdots & & & \vdots
\end{array}$$

Hierarchy Theorems for Deterministic Algorithms



Hierarchy Theorems for Deterministic Algorithms



Hierarchy Theorems for Nondeterministic Algorithms?

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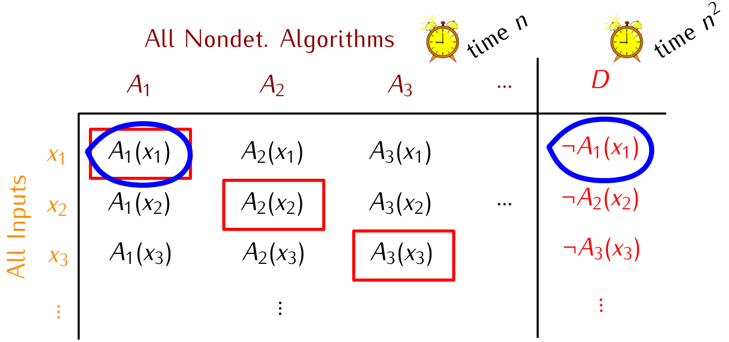

$$\begin{array}{c|cccc|c}
 & A_1 & A_2 & A_3 & \cdots & D \\
\hline
x_1 & A_1(x_1) & A_2(x_1) & A_3(x_1) & & \neg A_1(x_1) \\
x_2 & A_1(x_2) & A_2(x_2) & A_3(x_2) & \cdots & \neg A_2(x_2) \\
x_3 & A_1(x_3) & A_2(x_3) & A_3(x_3) & & \neg A_3(x_3) \\
\vdots & & \vdots & & & \vdots
\end{array}$$

Hierarchy Theorems for Nondeterministic Algorithms

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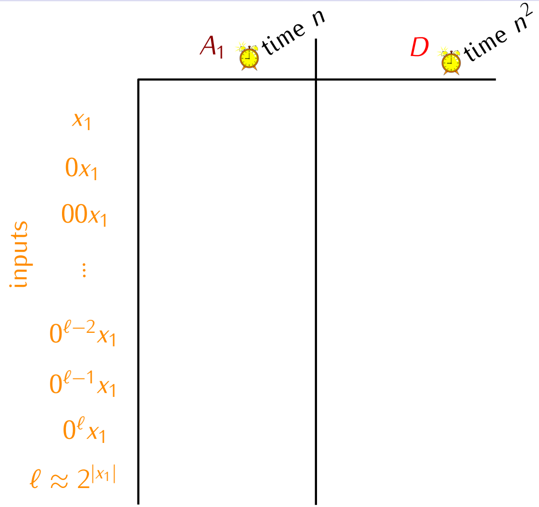
A_1 ⏰ time n

D ⏰ time n^2

Hierarchy Theorems for Nondeterministic Algorithms

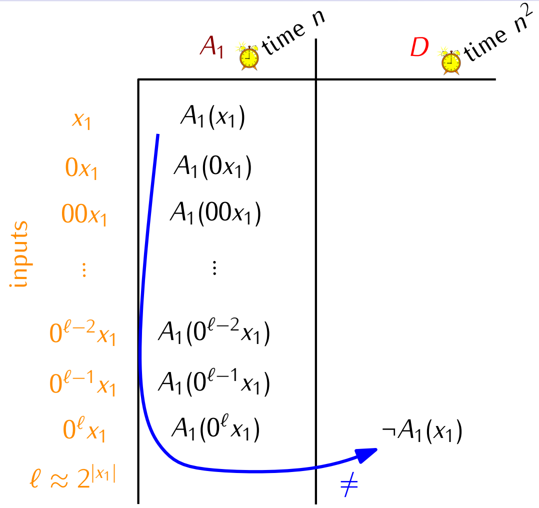


Hierarchy Theorems for Nondeterministic Algorithms

inputs

	A_1 time n	D time n^2
x_1	$A_1(x_1)$	
$0x_1$	$A_1(0x_1)$	
$00x_1$	$A_1(00x_1)$	
$\vdots$	$\vdots$	
$0^{\ell-2}x_1$	$A_1(0^{\ell-2}x_1)$	
$0^{\ell-1}x_1$	$A_1(0^{\ell-1}x_1)$	
$0^{\ell}x_1$	$A_1(0^{\ell}x_1)$	
$\ell \approx 2^{\lvert x_1 \rvert}$		

Hierarchy Theorems for Nondeterministic Algorithms



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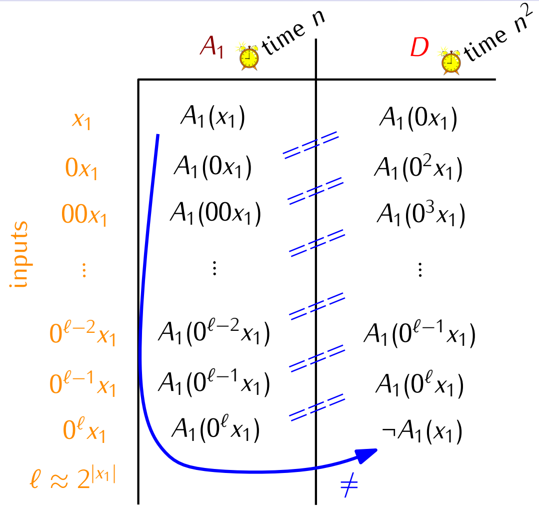
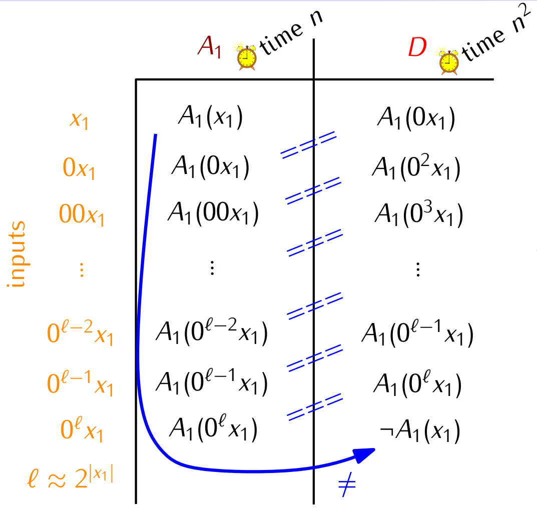
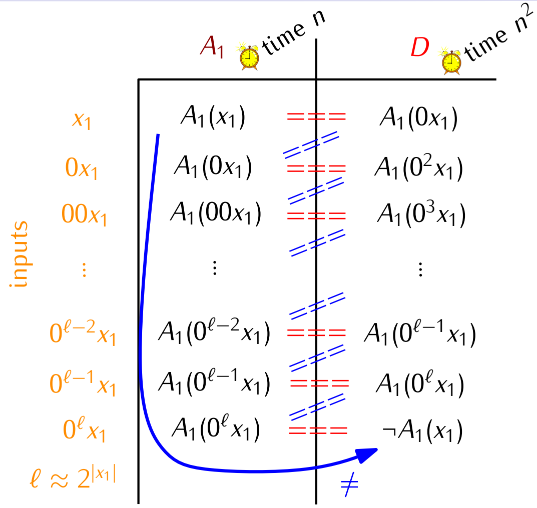


Hierarchy Theorems for Nondeterministic Algorithms



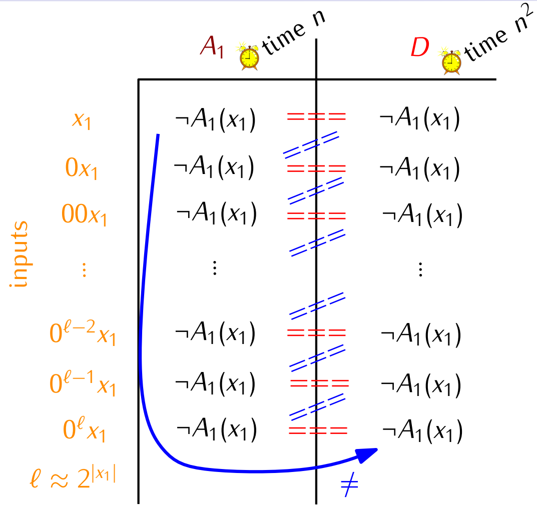
Assume A_1 the same as D on all inputs

Hierarchy Theorems for Nondeterministic Algorithms


A_1 time n
D time n^2
inputs
x_1
$0x_1$
$00x_1$
$0^{\ell-2}x_1$
$0^{\ell-1}x_1$
$0^{\ell}x_1$
$\ell \approx 2^{|x_1|}$
$A_1(x_1)$ $===$ $A_1(0x_1)$
$A_1(0x_1)$ $===$ $A_1(0^2x_1)$
$A_1(00x_1)$ $===$ $A_1(0^3x_1)$
$A_1(0^{\ell-2}x_1)$ $===$ $A_1(0^{\ell-1}x_1)$
$A_1(0^{\ell-1}x_1)$ $===$ $A_1(0^{\ell}x_1)$
$A_1(0^{\ell}x_1)$ $===$ $\neg A_1(x_1)$
$\neq$

Assume A_1 the same as D on all inputs

Hierarchy Theorems for Nondeterministic Algorithms



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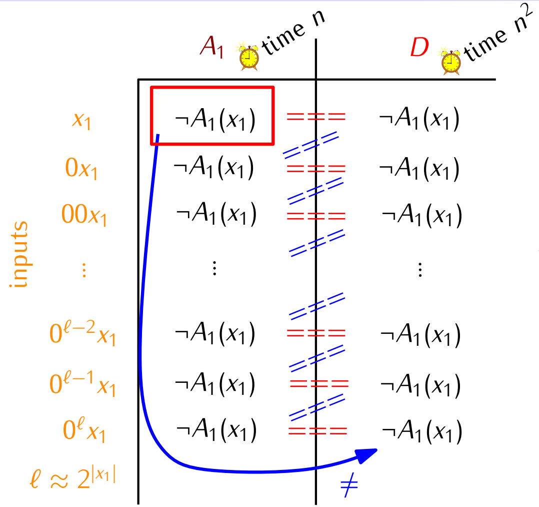
Hierarchy Theorems for Nondeterministic Algorithms



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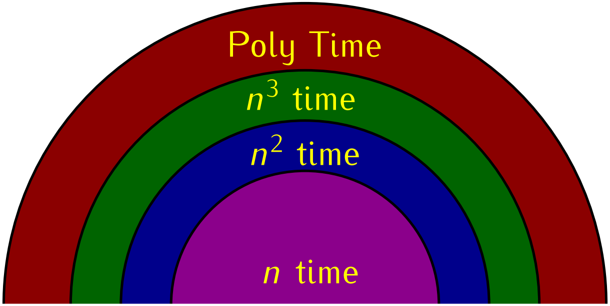
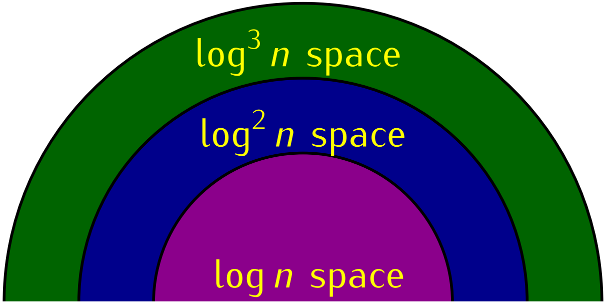


Hierarchy Theorems for Nondeterministic Algorithms



Hierarchy Theorems for Randomized Algorithms?

Hierarchy Theorems for Randomized Algorithms?

All Randomized Algorithms time n time n^2

All Inputs

$$\begin{array}{c|cccc|c}
 & A_1 & A_2 & A_3 & \cdots & D \\
\hline
x_1 & \boxed{A_1(x_1)} & A_2(x_1) & A_3(x_1) & & \neg A_1(x_1) \\
x_2 & A_1(x_2) & \boxed{A_2(x_2)} & A_3(x_2) & \cdots & \neg A_2(x_2) \\
x_3 & A_1(x_3) & A_2(x_3) & \boxed{A_3(x_3)} & & \neg A_3(x_3) \\
\vdots & & \vdots & & & \vdots
\end{array}$$

Hierarchy Theorems for Randomized Algorithms?

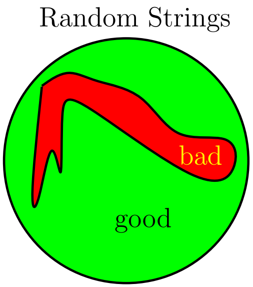
All Randomized Algorithms time n time n^2

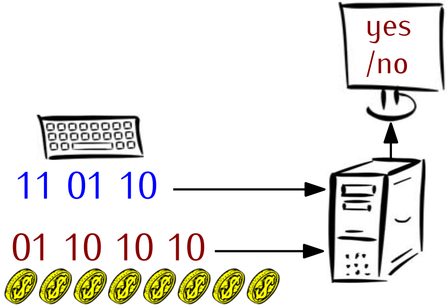
$$\begin{array}{c|cccc|c}
 & A_1 & A_2 & A_3 & \dots & D \\
\hline
x_1 & \boxed{A_1(x_1)} & A_2(x_1) & A_3(x_1) & & \neg A_1(x_1) \\
x_2 & A_1(x_2) & \boxed{A_2(x_2)} & A_3(x_2) & \dots & \neg A_2(x_2) \\
x_3 & A_1(x_3) & A_2(x_3) & \boxed{A_3(x_3)} & & \neg A_3(x_3) \\
\vdots & & \vdots & & & \vdots
\end{array}$$

All Inputs

- **What if** $\Pr[A_1(x_1) = \text{“yes”}] \approx .5$**?**

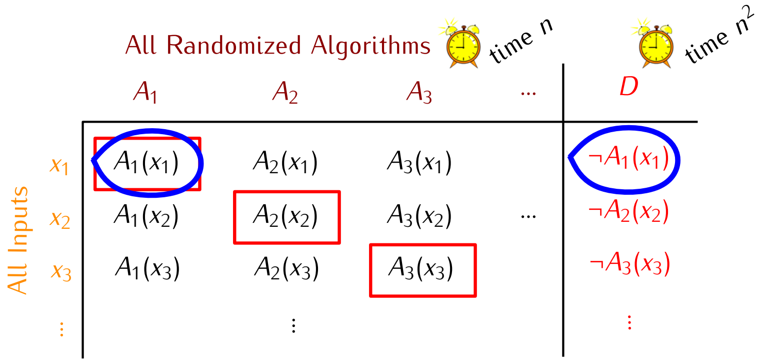
Randomized Algorithm





Bounded error: Correct with probability $> 99\%$

Hierarchy Theorems for Randomized Algorithms?


All Randomized Algorithms
time n
time n^2
A_1 A_2 A_3 ...
D
All Inputs
x_1 x_2 x_3 ⋮
$A_1(x_1)$ $A_2(x_1)$ $A_3(x_1)$
$A_1(x_2)$ $A_2(x_2)$ $A_3(x_2)$ ...
$A_1(x_3)$ $A_2(x_3)$ $A_3(x_3)$
$\neg A_1(x_1)$
$\neg A_2(x_2)$
$\neg A_3(x_3)$

- **What if** $\Pr[A_1(x_1) = \text{“yes”}] \approx .5$**?**

Hierarchy Theorems for Randomized Algorithms?

All Randomized Algorithms time n time n^2

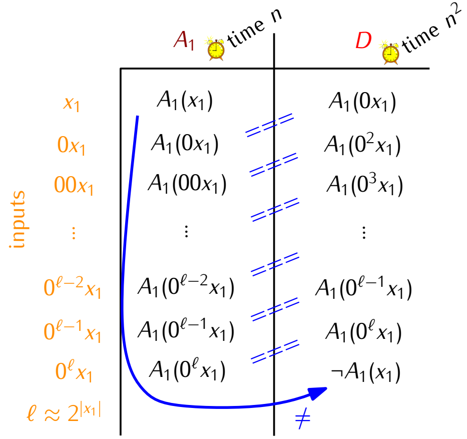
All Inputs

$$\begin{array}{c|cccc|c}
 & A_1 & A_2 & A_3 & \cdots & D \\ \hline
x_1 & \boxed{A_1(x_1)} & A_2(x_1) & A_3(x_1) & & \neg A_1(x_1) \\
x_2 & A_1(x_2) & \boxed{A_2(x_2)} & A_3(x_2) & \cdots & \neg A_2(x_2) \\
x_3 & A_1(x_3) & A_2(x_3) & \boxed{A_3(x_3)} & & \neg A_3(x_3) \\
\vdots & & \vdots & & & \vdots
\end{array}$$

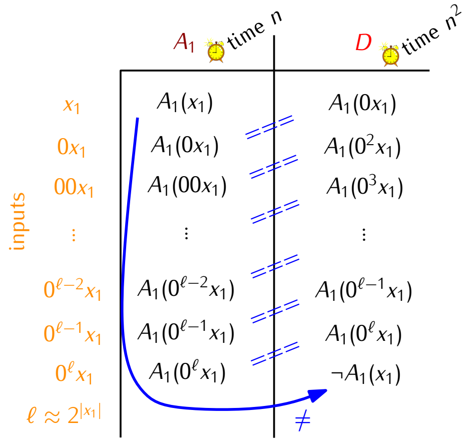
- **What if** $\Pr[A_1(x_1) =$ **"yes"**$] \approx .5$**?**
- Then D does not have bounded error, not valid

Hierarchy Theorems for Randomized Algorithms?

Hierarchy Theorems for Randomized Algorithms?

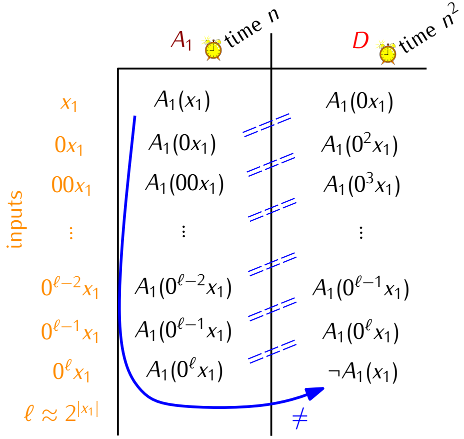


Hierarchy Theorems for Randomized Algorithms?



Make sure D has bounded error

Hierarchy Theorems for Randomized Algorithms?



Make sure D has bounded error – 1 bit of advice

Hierarchy Theorems for Randomized Algorithms?

Hierarchy Theorems for Randomized Algorithms?

- Yes, for algorithms with 1 bit of advice!

Hierarchy Theorems for Randomized Algorithms?

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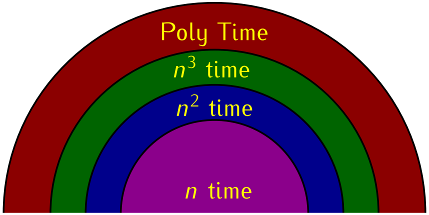

Hierarchy Theorems for Randomized Algorithms?

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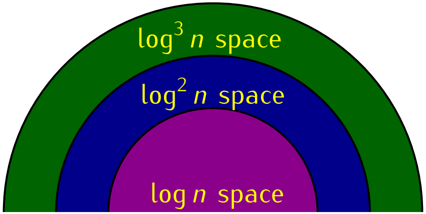

Hierarchy Theorems for Randomized Algorithms?

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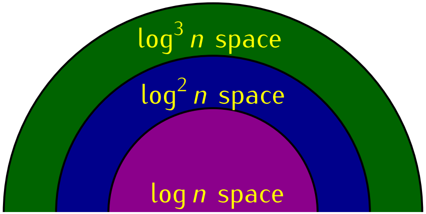

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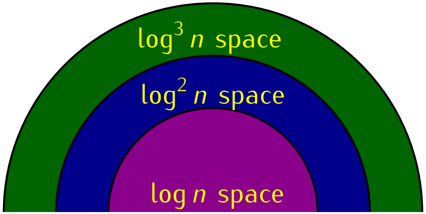

- My work [Kinne, Van Melkebeek]

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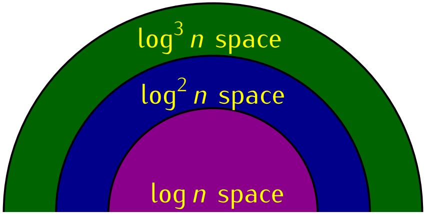

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  Memory Space hierarchies: randomized, quantum, ...

Computational Complexity Theory

How much **time, memory space, etc.** are needed to solve problems?

- **Is nondeterminism powerful?** P vs. NP
  - Conjecture: P$\neq$ NP
  - Techniques: hierarchy theorems, others
- **Is randomness powerful?** P vs. BPP, L vs. BPL
  - Conjecture: P=BPP, L=BPL
  - My work: derandomization, hierarchy theorems

The End, Thank You!

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Slides available at:
http://www.kinnejeff.com/GoSycamores/
(or E-mail me)

More on my research (slides, papers, etc.) at:
http://www.kinnejeff.com/