



# MIDWEST GRAPH THEORY CONFERENCE 52

April 27 - 28, 2012

Indiana State University

Terre Haute, Indiana

## **Organizing Committee:**

Robert Jajcay, Jeff Kinne, Yuejian Peng, and Geoffrey Exoo

## **Our sponsors:**

- College of Arts and Sciences, Indiana State University
- Department of Mathematics, Indiana State University
- Office of the President, Indiana State University
- Sonka Irish Pub, Terre Haute
- First Financial Bank

**Program:** All talks take place in the beautifully renovated

*University Hall, 401 North 7th Street, Terre Haute.*

All talks on Friday will be held in *Room 215* [R215]. Saturday talks will be held in Room 215 and the *Whitaker conference room* [WCR].

## **Invited Speakers:**

- *Gabriela Araujo-Pardo*, Instituto de Matematicas, Ciudad Universitaria, Mexico
- *Ted Dobson*, Mississippi State University
- *Tom Tucker*, Colgate University

## Schedule:

Thursday	7:30	Informal gathering at Sonka Irish Pub 1366 Wabash Avenue, Terre Haute
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Friday	1:30, front of [R215]	Registration
	2:00 - 2:15 [R215]	Official Opening of the Conference
	2:15 - 2:40 [R215]	Reza Akhtar: <i>Mod <math>n</math> representations of complete multipartite graphs</i>
	2:40 - 3:05 [R215]	Terry McKee: <i><math>k</math>-Partiteness of the Complements of Cographs</i>
	3:05 - 3:30 [R215]	Dalibor Fronček: <i>Magic rectangle sets and ordered distance antimagic graphs</i>
	3:30 - 4:00 [R215]	Coffee Break at Jazzman's Atrium of University Hall
	4:00 - 4:25 [R215]	David Anderson: <i>Animated Graph Theory</i>
	4:30 - 5:30 [R215]	Ted Dobson: <i>Vertex-transitive graphs</i>
	6:00	Dinner at George's Cafe 627 Cherry Street, Terre Haute
	8:00	Informal gathering at Sonka Irish Bar 1366 Wabash Avenue, Terre Haute

Saturday	[WCR]	[R215]
9:00 - 9:25	Xiangqian Zhou (Joe): <i>Clones in bicircular matroids</i>	Daniel McDonald: <i>List and Online List Variations of Vertex Ranking</i>
9:25 - 9:50	Hao-Hsiang Hung: <i>Light Spanners with Stack and Queue Charging Schemes</i>	Thomas Mahoney: <i>Extending graph choosability results to paintability</i>
9:50 - 10:30	Coffee Break at Jazzman's	
10:30 - 11:30	Tom Tucker: <i><math>K_7</math> in the torus: a long story</i>	
11:30 - 11:35	Break	
11:35 - 12:15	Ralph Grimaldi: <i>Extraordinary Subsets and a Partial Order</i>	Doug West: <i>Rainbow edge-coloring and rainbow domination</i>
12:15 - 1:30	Lunch at Jazzman's	
1:30 - 1:55	Mustafa Atici: <i>Secret sharing scheme and multipartite graphs</i>	Ben Reiniger: <i>Game Saturation Number of Graphs</i>
1:55 - 2:20	Matt Walsh: <i>Sliding colours in graphs</i>	Bill Kinnersley: <i>Rainbow Spanning Trees in Abelian Groups</i>
2:20 - 2:45	Chip Vandell: <i>Making the most of your decycling set</i>	Steve Butler: <i>Constructing universal graphs</i>
2:45 - 3:10	Coffee Break at Jazzman's	
3:10 - 4:10	Gabriela Araujo-Pardo: <i>Regular and bi-regular cages</i>	
4:10 - 4:35	David Galvin: <i>Colouring regular graphs without large independent sets</i>	Robert E. Jamison: <i>Long Cycles Represented by Paths</i>
4:35 - 5:00	John Engbers: <i>Counting independent sets of a fixed size in graphs with given minimal degree</i>	Ping Hu: <i>Upper bounds on the size of 4- and 6-cycle-free subgraphs of the hypercube</i>
5:00 - 5:25	John Rickert: <i>The Jacobsthal Subcube of the Hypercube</i>	Shariefuddin Pirzada: <i>On mark sequences of digraphs</i>

**Abstracts for the talks** listed alphabetically by the last name of the presenter:

### ***Mod $n$ representations of complete multipartite graphs***

**Reza Akhtar**

*Miami University*  
*email: akhtarr@muohio.edu*

A graph  $G$  has a representation modulo  $r$  if there exists an injective map  $f : V(G) \rightarrow \{0, 1, \dots, r-1\}$  such that vertices  $u$  and  $v$  are adjacent if and only if  $\gcd(f(u) - f(v), r) = 1$ ; the representation number  $rep(G)$  is the smallest positive integer  $r$  for which  $G$  has a representation modulo  $r$ . We describe some recent work (joint with Dan Pritikin and Tony Evans) on this parameter in the case that  $G$  is a complete multipartite graph.

### ***Animating Graph Theory***

**David J. Anderson**

*EdgeCase*  
*email: dave@edgecase.com*

Marrying the ubiquity of the web with the power of animation will help make Graph Theory more accessible and understandable to students, mathematicians, and the curious. This presentation provides a rationale for this perspective, shows technology that can perform web-based animation, and specifies the conceptual development of a system that will allow users to create, access, and reuse animatable Graph Theory algorithms.

### ***Regular and bi-regular cages***

**Gabriela Araujo-Pardo**

*Instituto de Matemáticas*  
*Universidad Nacional Autónoma de México*  
*email: garaujo@math.unam.mx*

In this talk, we give a brief summary of the Cage Problem and the relationship between cages of even girth that attain the Moore Bound and generalized polygons. Moreover, we investigate the same problem for bi-regular graphs.

We will present an overview of our results on these topics, and more specifically, the construction of regular and bi-regular graphs with small degree and fixed girth based on the use of geometric concepts and tools. We will emphasize that, in contrast to regular graphs, the bi-regular graphs we obtain using our constructions are best possible – bi-regular cages.

### ***Secret sharing scheme and multipartite graphs***

**Mustafa Atici**

*Western Kentucky University  
email: mustafa.atici@wku.edu*

Let  $\mathcal{P} = \{P_1, P_2, \dots, P_n\}$  be set of participant and  $\Gamma = \{B_i | B_i \subset \mathcal{P}, 1 \leq i \leq k\}$  be access structure. Vector space secret sharing scheme realizing access structure  $\Gamma$  requires existence of function  $\phi : \mathcal{P} \rightarrow (Z_p)^d$ , where  $p$  is a prime number and  $d \geq 2$  is an integer, satisfying the following condition:  $(1, 0, 0, 0, \dots, 0) = \langle \phi(P_i) : P_i \in B \rangle \Leftrightarrow B \in \Gamma = \{B_1, B_2, \dots, B_k\}$ . There is no known algorithm to construct such a function  $\phi$  in general. Constructions are mainly done by trial and error. In this talk, we give a polynomial algorithm to construct a  $\phi$  function for certain type of access structures. That is access structures that correspond edge set of multipartite graph  $G$ .

### ***Constructing universal graphs***

**Steve Butler**

*Iowa State University  
email: butler@iastate.edu*

Given a family of graphs we can work to construct the smallest graph which contains as induced subgraphs each graph in the family. We show that if we choose the family of all graphs on  $n$  vertices with bounded maximum degree that we can use some simple tools to construct near-optimal graphs. This construction also extends to multigraphs and digraphs.

## ***Vertex-transitive graphs***

**Ted Dobson**

*Mississippi State University and the University of Primorska  
email: dobson@math.msstate.edu*

A graph  $\Gamma$  is vertex-transitive if its automorphism group  $Aut(\Gamma)$  acts transitively on the vertex set  $V(\Gamma)$  of the graph. That is, if for every  $x$  and  $y$  in  $V(\Gamma)$ , there exists  $\psi$  in  $Aut(\Gamma)$  such that  $\psi(x) = y$ . Intuitively, a graph is vertex-transitive if it is not possible to distinguish between vertices. Many important graphs are vertex-transitive graphs (e.g. the Petersen graph, the Coxeter graph), and vertex-transitive graphs are important in chemistry and theoretical computer science, amongst other areas. Recently, vertex-transitive graphs received a fair amount of interest.

The purpose of this talk is to introduce what I consider some of the main problems (or perhaps just some of my favorite problems) in the study of vertex-transitive graphs, as well as indicate the kinds of results that have been and are currently being obtained concerning these problems. These problems include determining the full automorphism group of a vertex-transitive graph, determining necessary and sufficient conditions for two vertex-transitive graphs to be isomorphic, and Lovász's conjecture that every connected vertex-transitive graph contains a Hamilton path. By determining the automorphism group, we mean either an explicit list of groups, or a polynomial time algorithm to list a set of generators of the automorphism group. By "necessary and sufficient conditions for two graphs to be isomorphic" it is usually meant an explicit list  $L$  of maps, and two vertex-transitive graphs with a common minimal transitive subgroup are isomorphic if and only if they are isomorphic by a map on  $L$ .

## ***Counting independent sets of a fixed size in graphs with given minimal degree***

**John Engbers**

*University of Notre Dame  
email: jengbers@nd.edu*

Given a family of graphs, we can ask which graph in the family has the largest number independent sets, and we can also ask which graph in the family has the largest number of independent sets of a fixed size. Often, but not always, the extremal graphs that answer these two questions turn out to be the same. We will explore these questions within the family of graphs on  $n$  vertices with given minimal degree  $\delta$  and will show that the answer to these questions is almost always the complete bipartite graph  $K_{\delta, n-\delta}$ . We will also highlight some open questions. This is joint work with David Galvin.

## *Magic rectangle sets and ordered distance antimagic graphs*

**Dalibor Fronček**

*University of Minnesota Duluth*  
*email: dalibor@d.umn.edu*

Magic rectangles are a generalization of the well-known notion of magic squares. A *magic rectangle*  $MR(a, b)$  is an  $a \times b$  array whose entries are  $\{1, 2, \dots, ab\}$ , each appearing once, with all its row sums equal and with all its column sums equal. T. Harmuth proved in 1861 that a magic rectangle  $MR(a, b)$  exists if and only if  $a, b > 1$ ,  $ab > 4$ , and  $a \equiv b \pmod{2}$ .

We generalize this notion one step further by defining magic rectangle sets. A *magic rectangle set*  $MRS(a, b; c)$  is a collection of  $c$  arrays  $a \times b$  whose entries are  $\{1, 2, \dots, abc\}$ , each appearing once, with all row sums in every rectangle equal to a constant  $s$  and all column sums in every rectangle equal to a constant  $t$ .

We present constructions of some infinite classes of magic rectangle sets and explain our motivation for their study by showing a connection to incomplete handicap tournaments and ordered distance antimagic graphs.

An *ordered distance antimagic labeling* of a graph  $G(V, E)$  with  $n$  vertices is a bijection  $\vec{f} : V \rightarrow \{1, 2, \dots, n\}$  with the property that  $\vec{f}(x_i) = i$  and the sequence  $(w(x_1), w(x_2), \dots, w(x_n))$  forms an increasing arithmetic progression with difference one where  $w(x_i)$ , the *weight* of vertex  $x_i$ , is the sum of the labels of all neighbors of  $x_i$ . A graph  $G$  is an *ordered distance antimagic graph* if it allows an ordered distance antimagic labeling.

## *Colouring regular graphs without large independent sets*

**David Galvin**

*University of Notre Dame*  
*email: galvin1@nd.edu*

Among all  $n$ -vertex,  $d$ -regular graphs, which one admits the most proper  $q$ -colourings for each  $q$ ? There's an appealing conjecture, that when  $2d$  divides  $n$  the answer is the same for every  $q$ : it's the disjoint union of  $n/2d$  copies of  $K_{d,d}$ , the complete bipartite graph with  $d$  vertices in each partite set.

If we restrict to bipartite graphs, the conjecture is true. For regular graphs which are close to being bipartite, in the sense that they have an independent set consisting of roughly half the vertices, an asymptotic version of the conjecture is known.

Here we look at the other side of the picture. We put an upper bound on the number of  $q$ -colourings admitted by a regular graph in terms of the independence number of the graph, and so show that the conjecture concerning  $K_{d,d}$ 's is true for every graph that is not close to being bipartite.

## ***Extraordinary Subsets and a Partial Order***

**Ralph P. Grimaldi**

*Rose-Hulman Institute of Technology*

*email: grimaldi@rose-hulman.edu*

For a positive integer  $n$ , a subset  $S$  of  $[n] = \{1, 2, \dots, n\}$  is called *extraordinary* when the minimal element in  $S$  is the same as the size of  $S$ . If we let  $a_n$  count the number of extraordinary subsets of  $[n]$ , we find that  $a_n = F_n$ , the  $n$ -th Fibonacci number. Following this, we examine  $a(n, k)$ , the number of times the positive integer  $k$  appears among the  $a_n$  extraordinary subsets of  $[n]$ . Then we look to

- (i)  $t_n$ , which counts the total number of elements (repeats are counted) that occur among the  $a_n$  extraordinary subsets of  $[n]$ ; and,
- (ii)  $s_n$ , which is the sum of all the elements (repeats are counted) that occur among the  $a_n$  extraordinary subsets of  $[n]$ .

Finally, for a given  $n$ , we introduce a partial order for the  $a_n$  subsets of  $[n]$  and determine a formula for the number of edges that occur in the Hasse diagram for the partial order.

## ***Upper bounds on the size of 4- and 6-cycle-free subgraphs of the hypercube***

**Ping Hu**

*University of Illinois at Urbana-Champaign*

*email: pinghu1@illinois.edu*

Erdős proposed the problem of determining  $ex_Q(n; C_{2t})$ , i.e. to determine the maximum number of edges that a subgraph of the  $n$ -dimensional hypercube containing no  $C_{2t}$  can have.

We modify slightly Razborov's flag algebra machinery to be suitable for the hypercube. We use this modified method to show that the maximum number of edges in a subgraph of the  $n$ -dimensional hypercube containing no 4-cycle is at most 0.6068 times the number



of edges in the hypercube. For subgraphs containing no 6-cycle, we improve the upper bound on the proportion of edges from  $\sqrt{2} - 1$  to 0.3755.

(Joint work with Jozsef Balogh, Bernard Lidicky, and Hong Liu.)

## ***Light Spanners with Stack and Queue Charging Schemes***

**Hao-Hsiang Hung**

*Emory University*

*email: hhung2@emory.edu*

We consider the problem of decomposing light spanners in some minor-closed graph families. We provide an approach employing two charge patterns. That is, given a spanning tree  $T$  within a light spanner, we partition the remaining edges into a bounded number of classes, each of which is either "stack like" or "queue like" with respect to an Euler tour of  $T$ . We show that this approach works well for bounded genus graphs and could attack subfamilies of bounded genus graphs, but is not enough for the general case. We could counterexamples for such a mixed scheme

## ***Long Cycles Represented by Paths***

**Robert E. Jamison and Douglas B. West**

*University of Illinois, Urbana-Champaign*

*email: rejamis@illinois.edu, west@math.uiuc.edu*

Many graph classes are defined by the following paradigm: each vertex is assigned something measuring its size and something measuring its tolerance. If the combined sizes exceed the combined tolerances, then there is a *conflict* and the corresponding vertices are adjacent in a *conflict-tolerance graph*. In this talk we construct such representations for long chordless cycles in the case that the "sizes" are paths  $P_r$  in  $K_n$  and "conflict" occurs when two paths contain a common  $P_q$  for some fixed  $q < r$ .

## ***Rainbow spanning trees in Abelian groups***

**Bill Kinnersley**

*University of Illinois at Urbana-Champaign*

*email: wkinner2@illinois.edu*

Given an additive Abelian group  $A$ , let  $K_A$  denote the edge-colored complete graph in which the vertices are the elements of  $A$ , and edge  $xy$  has color  $x + y$ . Which trees appear as rainbow subgraphs in  $K_A$  (that is, subgraphs with no two edges of the same color)? This question naturally gives rise to a labeling problem with close ties to harmonious, elegant, and cordial labeling.

In this talk, we provide some necessary and some sufficient conditions for a tree  $T$  to embed as a rainbow spanning tree in  $K_A$ . When  $A = \mathbf{Z}_n$ , a conjecture due to Hovey would imply that every  $n$ -vertex tree embeds as a rainbow spanning tree in  $K_A$ ; we show that the analogous statement fails whenever  $A$  is non-cyclic. With computer assistance, for every Abelian group  $A$  of order at most 20, we determine all trees on  $|A|$  vertices that fail to embed as rainbow spanning trees in  $K_A$ . (There are not very many!)

This is joint work with Robert E. Jamison.

## ***Extending graph choosability results to paintability***

**Thomas Mahoney**

*University of Illinois at Urbana-Champaign*  
*email: tmahone2@illinois.edu*

Introduced independently by Schauz and by Zhu, the *Marker/Remover* game is an on-line version of list coloring. The resulting graph parameter, *paintability*, is at least the chromatic number (the *choosability*). We discuss the extension of various choosability results to paintability. These include the analogue of Ohba's conjecture, bounds on the paintability of complete bipartite graphs, characterization of 3-paint-critical graphs, and equality of paintability and chromatic number for claw-free perfect graphs with no 4-clique. In more detail, we then introduce and study *sum-paintability*, the analogue of sum-choosability.

## ***List and Online List Variations of Vertex Ranking***

**Daniel McDonald**

*University of Illinois at Urbana-Champaign*  
*email: dmcdona4@illinois.edu*

A vertex ranking of a graph  $G$  is a labeling of its vertices with positive integers such that any path whose endpoints have the same label contains a larger label; applications of rankings include VLSI design, parallel computing, and factory scheduling. The list rank number of  $G$  is the least positive integer  $k$  such that if each vertex of  $G$  is assigned a set

of  $k$  potential labels,  $G$  can always be ranked by labeling each vertex with a label from its assigned list. We formulate an online version of the list rank number and compute the online list rank number of paths and cycles as well as the list rank number of trees with many more leaves than internal vertices.

### ***k-Partiteness of the Complements of Cographs***

**Terry McKee**

*Wright State University*  
*email: terry.mckee@wright.edu*

A graph is a cograph if and only if it reduces to an edgeless graph by repeatedly taking complements within components, which is known to be equivalent to every induced subpath having at most two edges. I discuss various sorts of characterizations of those nontrivial connected cographs for which all the nontrivial components of the complement are complete  $k$ -partite. For instance, one of several characterizations for the bipartite case is that every induced subtree is a path with at most two edges.

### ***The Jacobsthal Subcube of the Hypercube***

**John Rickert**

*Rose-Hulman Institute of Technology*  
*email: john.rickert@rose-hulman.edu*

We study a special subset of the  $n$ -dimensional hypercube, the Jacobsthal subcube, defined recursively by only including strings of 0s and 1s that can be written by combining the strings 0, 01, and 11. The number of vertices in the subcube is given by the  $n$ -th Jacobsthal number  $J_n$ , defined by the Fibonacci-like recurrence relation  $J_n = J_{n-1} + 2J_{n-2}$ ,  $J_0 = 1$ ,  $J_1 = 1$ . We count the number of edges in the Jacobsthal subcube and discover Hamiltonian paths. This is joint work with Ralph Grimaldi and Tom Langley.

### ***$K_7$ in the torus: a long story***

**Thomas Tucker**

*Colgate University, Hamilton, New York*  
*email: ttucker@colgate.edu*

The embedding of  $K_7$  in the torus has played an important role in topological graph theory. It is, of course, the first step in the Heawood Map Color Theorem, but it also inspired current maps, voltage graphs, rotation systems, Cayley maps, and chirality for regular maps. We will try to trace out parts of the story, including a little known tale from Jack Edmonds about how a conjecture in Coxeter and Moser about chiral maps led him to rotation systems. We will finish with an absurdly simple proof that regular (reflexible) maps have clique number 2,3,4, or 6, a result previously obtained only through elaborate algebraic arguments.

### *Sliding colours in graphs*

**Landan Hicks, Marc Lipman, and Matt Walsh\***

*Indiana University - Purdue University Fort Wayne  
email: walshm@ipfw.edu*

The vertices of a graph  $G$  are coloured improperly, but in a way such that some permutation of the colouring will be proper. We want to restore the proper colouring; we can do so by removing the colours on some vertices (creating “holes” in the colouring) and sliding colours from vertices into adjacent holes. What is the minimum number of holes required to restore any such scrambled colouring? We give partial answers to these questions, and discuss some variations on the theme.

### *Rainbow edge-coloring and rainbow domination*

**Douglas B. West**

*University of Illinois at Urbana-Champaign  
email: west@math.uiuc.edu*

Let  $G$  be an edge-colored graph with  $n$  vertices. A *rainbow subgraph* is a subgraph whose edges have distinct colors. The *rainbow edge-chromatic number* of  $G$ , written  $\hat{\chi}'(G)$ , is the minimum number of rainbow matchings needed to cover  $E(G)$ . An edge-colored graph is *t-tolerant* if it contains no monochromatic star with  $t + 1$  edges. If  $G$  is  $t$ -tolerant, then  $\hat{\chi}'(G) < t(t+1)n \ln n$ , and examples exist with  $\hat{\chi}'(G) \geq \frac{t}{2}(n-1)$ . The *rainbow domination number*, written  $\hat{\gamma}(G)$ , is the minimum number of disjoint rainbow stars needed to cover  $V(G)$ . For  $t$ -tolerant edge-colored  $n$ -vertex graphs, we generalize classical bounds on the domination number:

- (1)  $\hat{\gamma}(G) \leq \frac{1+\ln k}{k}n$  (where  $k = \frac{\delta(G)}{t} + 1$ ), and
- (2)  $\hat{\gamma}(G) \leq \frac{t}{t+1}n$  when  $G$  has no isolated vertices.

We also characterize the edge-colored graphs achieving equality in the latter bound.

This joint work with Timothy D. LeSaulnier.

## *Clones in Bicircular Matroids*

**Xiangqian Zhou (Joe)**

*Department of Mathematics and Statistics, Wright State University*

*email: xiangqian.zhou@wright.edu*

There are two fundamental classes of matroids related to graphs: the well-known one is the class of graphic matroids where a circuit of the matroid is the edge set of a cycle in the graph; the other one is the class of bicircular matroids where a circuit of the matroid is the edge set of a minimal connected subgraph containing at least two cycles of the graph.

Two elements in a matroid are clones if the map that interchanges the two and fixes all other elements is an automorphism of the matroid. Clones have recently become an interesting subject in matroid representation theory. In the talk, we will describe exactly when two elements of a bicircular matroid form a clonal pair. This is joint work with Daniel Slilaty and Jakayla Robbins.