Light Spanners with Stack and Queue Charging Schemes

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Outline

Motivation

- Metrical Optimization Problems in Graphs (e.g. TSP)
- Previous Work: Charging Schemes
- Book Embedding vs. Stack and Queue Charging Scheme
- Graph Families for Queue and Stack Schemes

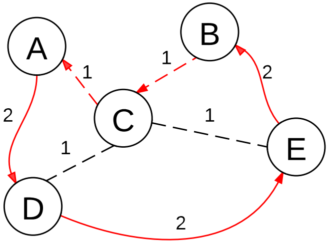
Results for the Talk

- Charging Schemes for Bounded Pathwidth Graphs

Outline

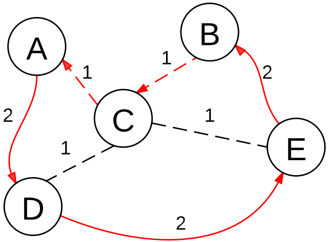
Traveling Salesman Problem

- TSP – NP Complete
- 1-2 TSP – MAX-SNP Hard
- Metric TSP – ∃ A Fast 2 Approximation Algorithm



Metric TSP

- There are approximation algorithms for Metric TSP with bounded errors.
- Have: Error $\leq \epsilon w(G)$
- Want: Error $\leq \epsilon w(MST)$
- Lucky: $w(G') \leq \epsilon w(MST)$ G': pruned graph from G

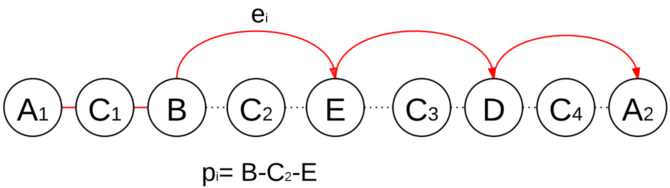


Light Spanners for Metric Optimization

- Candidate: *Light Spanners*
- $G' = Span(G, 1+\epsilon)$ with the following good properties:

1 "Span": for $u, v \in V$, $d_{G'}(u,v) \leq (1+\epsilon) d_G(u,v)$

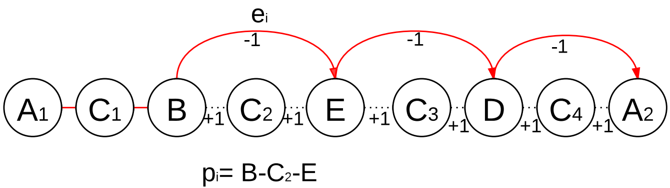
2 "Light": $w(G') \leq \frac{k}{\epsilon} w(MST)$



Outline

Charging Scheme

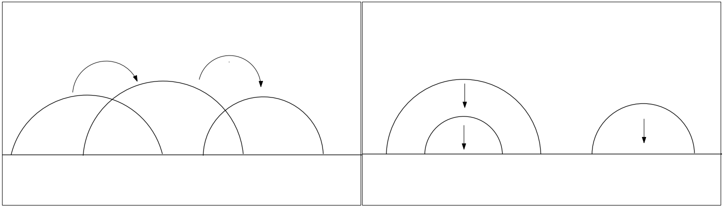
- Charging Scheme (Proved by LP duality)
- For each (e_i, p_i), e_i pay 1 unit of charge, every $e \in p_i$ receive 1 unit of charge
- Goal of the Dual Problem: to minimize the value of charges received for edges of trees



Outline

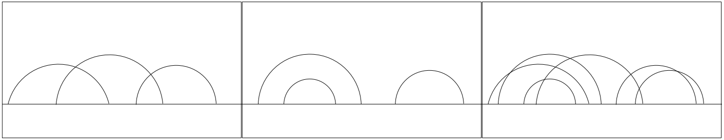
Book Embedding vs. Charging Schemes

- Book Embedding: A book drawing of G onto a book B should be:
  - every vertex of G is mapped to the spine of B; and
  - every edge of G is mapped to a single page of B.
- A book embedding of G onto B requires the drawing does not have crossings.
- Every page is (outer)-planar
- Queue Scheme/Queue-compatible Page
- Stack Scheme/Stack-compatible Page



Queue and Stack Charging Schemes

- (c, d)-graph
- c – Number of Queue Pages
- d – Number of Stack Pages
- Retrospect: "Light": $w(G') \leq \frac{k}{\epsilon} w(MST)$
- $k = 2c + d$
- If c, d are $O(1) \rightarrow k$ is, too.



Outline

Previous Work

- Planar Graphs $\rightarrow (0, 2)$-graphs
- Technique: No Crossing
- Bounded Genus Graphs $\rightarrow (6g - 2, 3g - 2)$-graphs
- Technique: Decompose Bounded Genus Graphs into union of planar graphs

Graph Minor Theory

- Robertson-Seymour Theory: graphs of minor-closed family can be decomposed into the following components:
  1. Bounded Genus Graphs
  2. Apices
  3. Vortices
  4. Clique Sums
- Vortices: Bounded Pathwidth Graphs stitched to the surface
- Grigni's conjecture: every minor close graph family has light spanners

Charging Bounded Pathwidth Graphs

- To charge Bounded Pathwidth Graphs:
  1. Convert it to Bounded Bandwidth Graphs
  2. Construct a path by taking an Euler Tour of MST
  3. Assume MST is a path, we show a counterexample
  - $\hat{G} \rightarrow (O(\sqrt{n}), O(\sqrt{n}))$-graphs and Bounded Pathwidth

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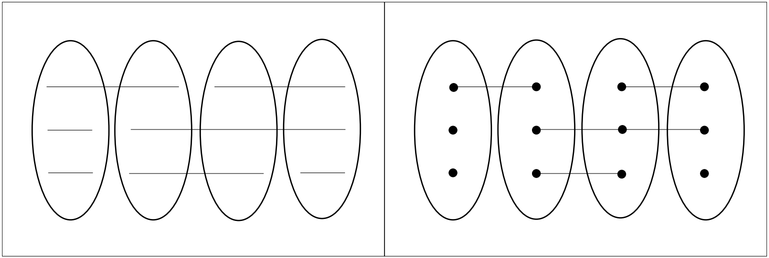
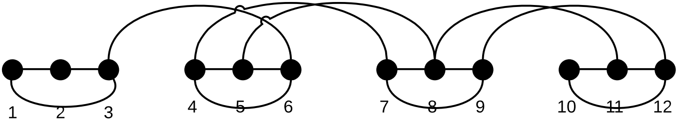
Convert Bounded Pathwidth Graphs to Bounded Bandwidth Graphs



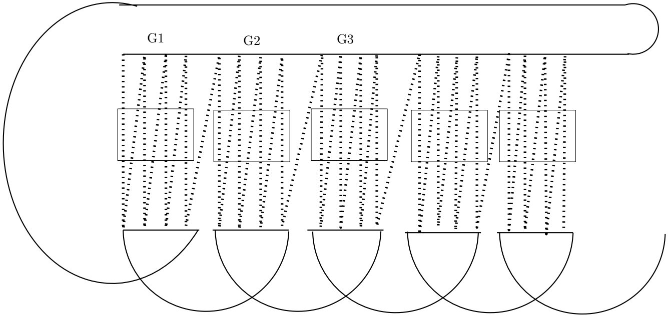
Bounded Bandwidth Graphs

- Goal: To Bound the Maximum Degree
- Assume weight 0 to edges between duplicate vertices



Bounded Pathwidth Graphs: Counterexample

- Solid Line: the MST T of G'
- Zig-Zag Line: edges not in T ($e \in G' - T$)
- $O(\sqrt{n})$ Zig-Zag Edges in each group; total $O(\sqrt{n})$ groups



Summary

- Queue and Stack charging scheme cannot handle bounded pathwidth graphs
- However, we are able to solve it by creating a structure called "monotone tree" (http://arxiv.org/abs/1104.4669)

- Future Work
  - How to connect vortices to the plane or bounded genus graphs?
  - How to handle clique sum individually?