Graphic Representations of Long Chordless Cycles

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host graph H where the representation occurs

target graph G that we wish to represent

We assign a representative subgraph R_v of H to each vertex v in G
so that v and w are adjacent iff R_v and R_w "conflict" —
that is, they have "enough in common".

Examples:

Host: A tree T
Targets: Chordal graphs
Representatives: Subtrees of T
"Enough" (Conflict rule): A node

NOTE: Vertices of the target are "vertices"
Vertices of the host are "nodes".

In this talk, all representatives will be isomorphic to a fixed **representative prototype**.

The conflict rule kicks in when two representatives contain a common copy of a fixed **quota**.

Host: A graph H
Targets: Line graphs
Representatives: P_2 — i.e., edges of H
Quota: P_1 — i.e., a node of H.

Representative prototype: P_4
Quota prototype: P_2

r: order of the representative prototype R
q: order of the quota prototype Q

An $(H; R, Q)$-*representation* of a graph G in a host H is an injective assignment $v \to R_v$ of a copy R_v of R to each vertex v of the target such that

$$vw \in E(G)$$
$$\Longleftrightarrow$$
$$R_v \cap R_w \text{ contains a copy of } Q.$$

The *universal graph* has all copies of R in H as vertices, with adjacency determined as above.

G is representable $\Longleftrightarrow$
G an induced subgraph of the universal graph

Let K_n be the host.

Let M be the maximum number of Q-copies contained in an induced subgraph of H of order q.

Let G be the target graph with maximum clique size ω.

The order of G is bounded by $\omega M\binom{n}{q}$.

M and ω are constants with respect to n, so the maximum order of representable graphs with bounded clique size is $O(n^q)$.

This order is achieved if the representative prototype is P_r and the quota is P_q.

Assumptions:
$r > q \geq 2$
For a fixed $m \geq 2$ assume
$r - q$ and r both divide m^q.

Select a de Bruijn sequence a_k on an alphabet of m letters
such that each q-tuple occurs exactly once.
Take the alphabet to be $[m]$.

Reference: Robert E. Jamison, Towards a Comprehensive Theory of Conflict-Tolerance Graphs, Proceedings of LAGOS Conference, to appear in Discrete Applied Math.