Rainbow spanning trees in Abelian groups

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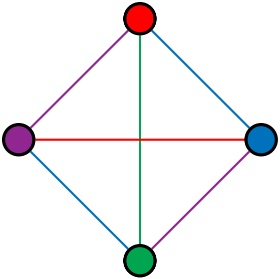
Joint work with

Robert E. Jamison

Rainbow spanning trees

Label the vertices of K_n with elements of $\mathbb{Z}_n$;
label each edge with the sum of its endpoints.



$\mathbb{Z}_4$:

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Which 4-vertex trees appear as rainbow trees?

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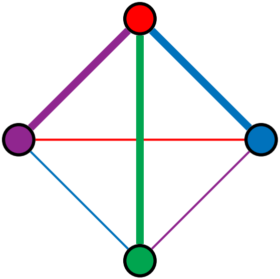

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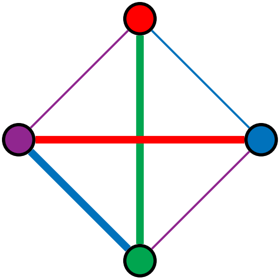

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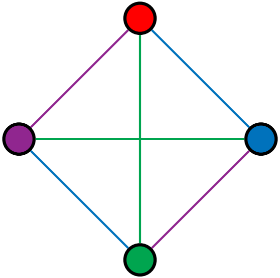

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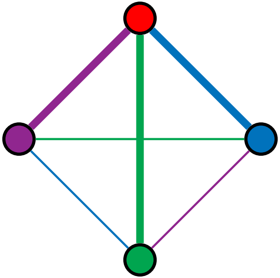

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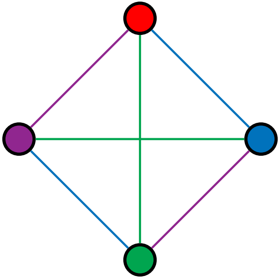

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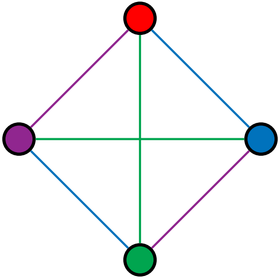

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Given an Abelian group A, let K_A denote the corresponding edge-colored complete graph.

Which trees appear as rainbow spanning trees in K_A?

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Prior work: Beals-Gallian-Headley-Jungreis [cycles], Valentin [paths, cycles], Zheng [$A = \mathbb{Z}_2^k$]

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Cordial labeling:
Label from Abelian group A.
Distribution of labels on vertices is balanced.
So is distribution of sums on edges.

Cyclic groups

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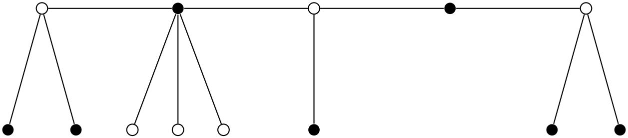

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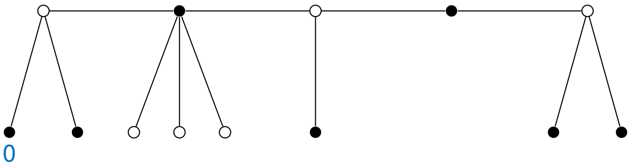

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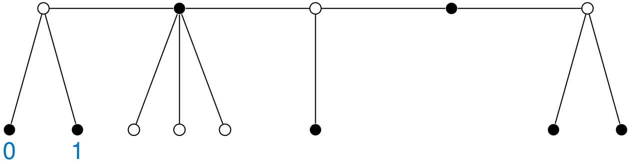

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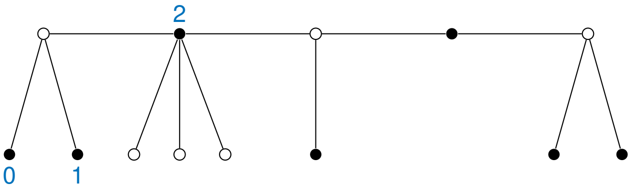

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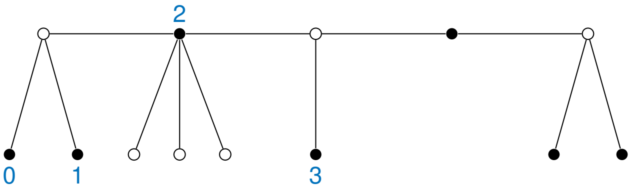

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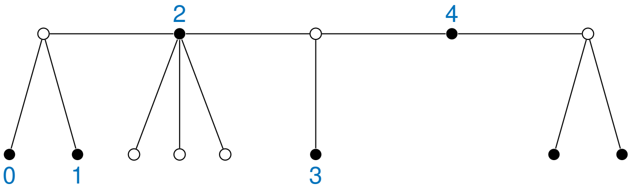

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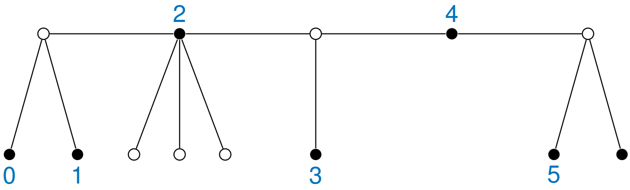

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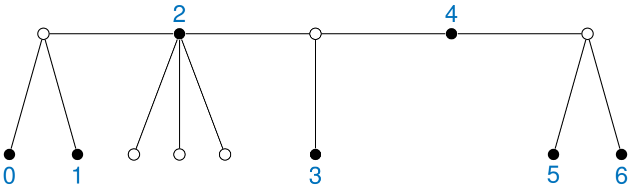

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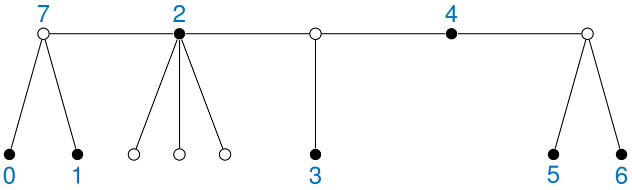

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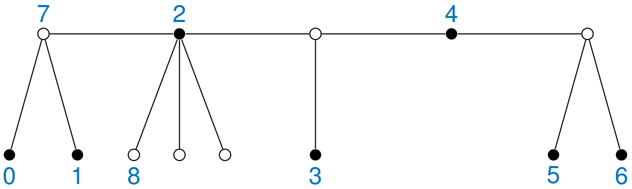

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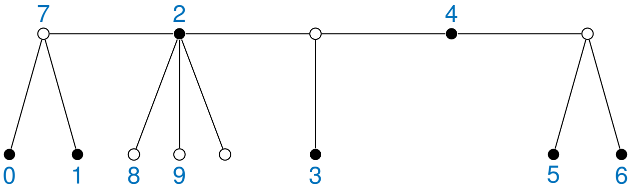

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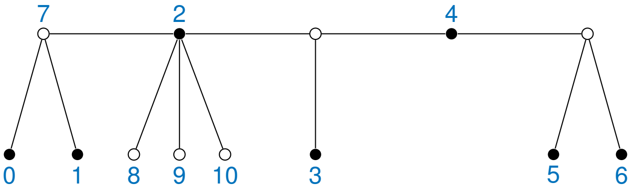

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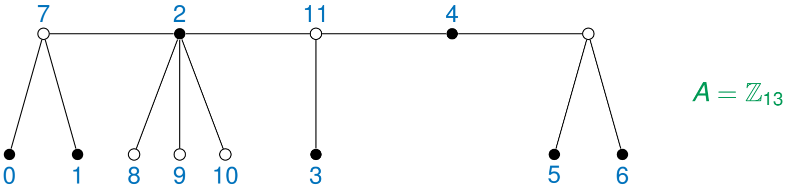

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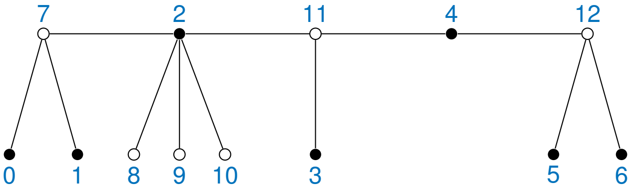

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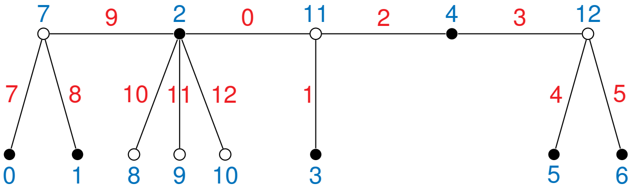

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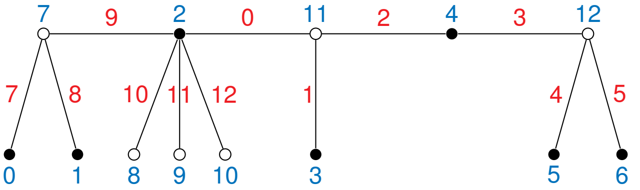

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Conjecture (Hovey)

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Theorem

Let A be an Abelian group with order n and characteristic m.
Let T be an n-vertex tree. If T has vertices u and v such that:

- $d(u) \equiv d(v) \equiv 0 \pmod{m}$,
- $d(x) \equiv 1 \pmod{m}$ *for all* $x \in V(T) - \{u, v\}$, *and*
- $uv \in E(T)$,

then T is not A-iridescent.

(Recall: the characteristic of A is the least m such that $ma = 0$ for all $a \in A$.)

Non-iridescence

Corollary

Let A be an Abelian group with order n and characteristic m. Let T be a tree with at least two vertices. If $n \geq m|V(T)|$, then T is contained in an n-vertex tree that is not A-iridescent.

(Note: the condition that $n \geq m|V(T)|$ forces A to be non-cyclic.)

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On the other hand:

Proposition

Let A be an Abelian group of order n and let T be a tree. If $n \geq 2|V(T)| - 2$, then T is contained in some n-vertex A-iridescent tree.

Thus iridescence is not a “local” property.

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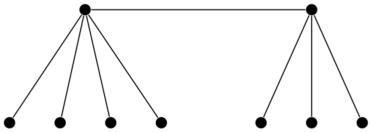

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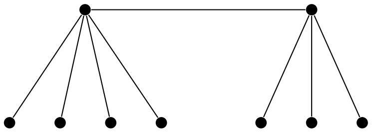

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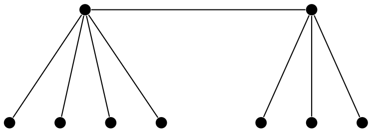

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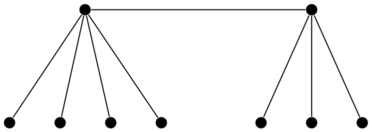

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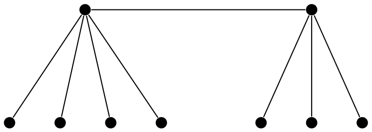
T satisfies our earlier condition for non-iridescence.



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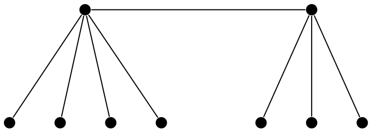
Proof.

Now suppose $k \not\equiv -1 \pmod{m}$.
Choose $a \in A$ with order m; partition A into cosets of $\langle a \rangle$.



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cosets: $\{00, 10, 20\}$
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Small caterpillars

$C[h_1, \ldots, h_s]$: the caterpillar with s spine vertices, where the ith spine vertex has h_i pendant leaves.

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Let A be an Abelian group with order n and characteristic m. An n-vertex caterpillar T of the form $C[k, \ell]$ is A-iridescent iff $k \not\equiv -1 \pmod{m}$.

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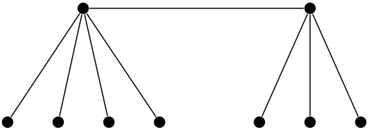

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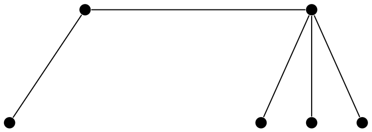

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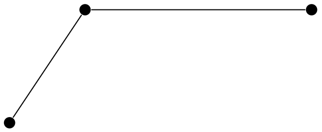

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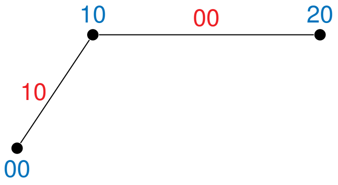

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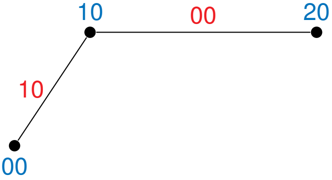

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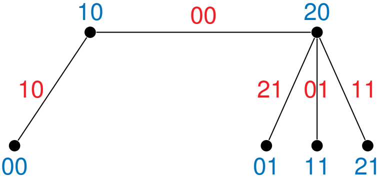

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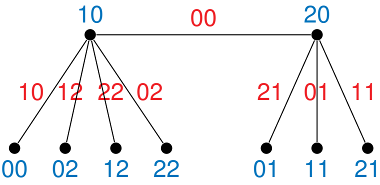

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Theorem

Let A be an Abelian group with order n and characteristic m.
Let T be an n-vertex caterpillar of the form $C[k, 0, \ell]$.
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Theorem

Let A be an Abelian group with order n and characteristic m.
Let T be an n-vertex caterpillar of the form $C[k, 0, 0, \ell]$.
T fails to be A-iridescent iff either

- *$k \equiv -2 \pmod{m}$, or*
- *$T = C[n - m - 1, 0, 0, m - 3]$ and $A = \mathbb{Z}_m^k$ for $k \geq 2$ and m an odd prime.*

Non-iridescent trees

Computer-aided search for non-iridescent trees:

- checked all Abelian groups of order at most 20
- for each group, checked all trees of same order

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Most of the ones we found are $C[k,\ell]$, $C[k,0,\ell]$, or $C[k,0,0,\ell]$.

Didn't find any counterexamples to Hovey's conjecture on cyclic groups.
Didn't touch Boolean groups $\mathbb{Z}_2^k$; Zheng has those covered.

Non-iridescent trees

Order 8: $\mathbb{Z}_4 \times \mathbb{Z}_2$

small caterpillars: $C[3,3]$ $C[2,0,3]$ $C[2,0,0,2]$

Non-iridescent trees

Order 9: $\mathbb{Z}_3 \times \mathbb{Z}_3$

small caterpillars:	$C[2,5]$	$C[1,0,5]$ $C[2,0,4]$	$C[1,0,0,4]$
misc. explained:	$C[2,1,3]$	$C[1,1,0,3]$	$C[2,0,0,0,2]$

Non-iridescent trees

Order 12: $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

small caterpillars: $C[5,5]$ $C[5,0,4]$ $C[4,0,0,4]$

Non-iridescent trees

Order 16: $\mathbb{Z}_8 \times \mathbb{Z}_2$

small caterpillars: $C[7,7]$ $C[6,0,7]$ $C[6,0,0,6]$

Non-iridescent trees

Order 16: $\mathbb{Z}_4 \times \mathbb{Z}_4$

small caterpillars:	$C[7,7]$	$C[6,0,7]$	$C[6,0,0,6]$
	$C[3,11]$	$C[2,0,11]$	$C[2,0,0,10]$
		$C[10,0,3]$	

---

misc. explained:

$C[3,2,8]$
$C[3,6,4]$
$C[7,2,4]$
$C[3,2,3,4]$
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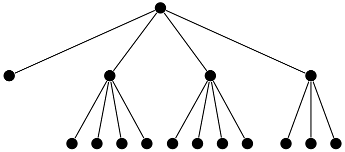

Non-iridescent trees

Order 16: $\mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

small caterpillars: $C[7,7]$ $C[3,11]$ $C[6,0,7]$ $C[2,0,11]$ $C[10,0,3]$ $C[6,0,0,6]$ $C[2,0,0,10]$

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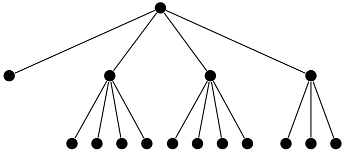

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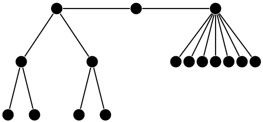

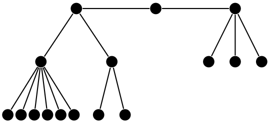

Non-iridescent trees

Order 18: $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_2$

small caterpillars:	$C[5, 11]$	$C[4, 0, 11]$ $C[5, 0, 10]$	$C[10, 0, 0, 4]$

---

misc. explained: $C[5, 4, 6]$

Non-iridescent trees

Order 20: $\mathbb{Z}_5 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

small caterpillars: $C[9,9]$ $C[8,0,9]$ $C[8,0,0,8]$

Future work

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- “unexplained” trees for $\mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$
- more *sufficient* conditions for iridescence
- general caterpillars

Thanks

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