k-Partiteness of the Complements of Cographs

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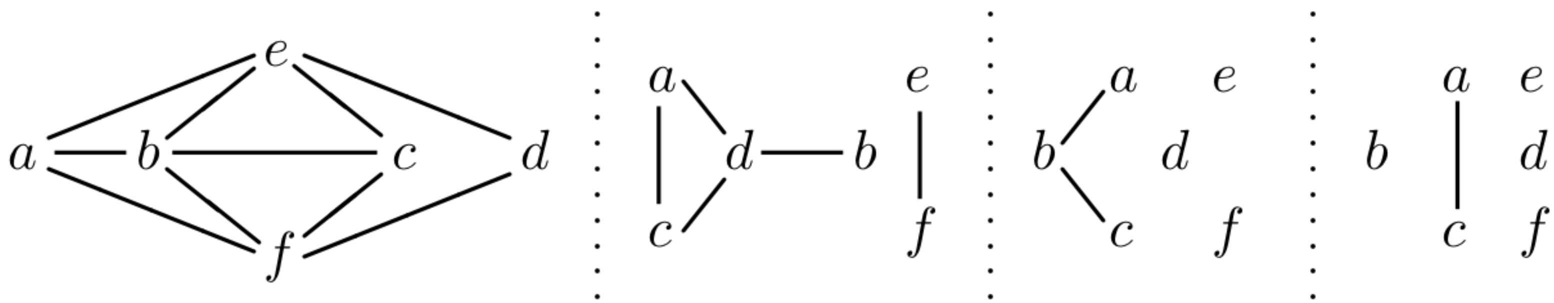
MIGHTY 52 Indiana State University April 27, 2012

A graph is a cograph (= complement-reducible graph) if it can be reduced to an edgeless graph by repeatedly taking complements within components.



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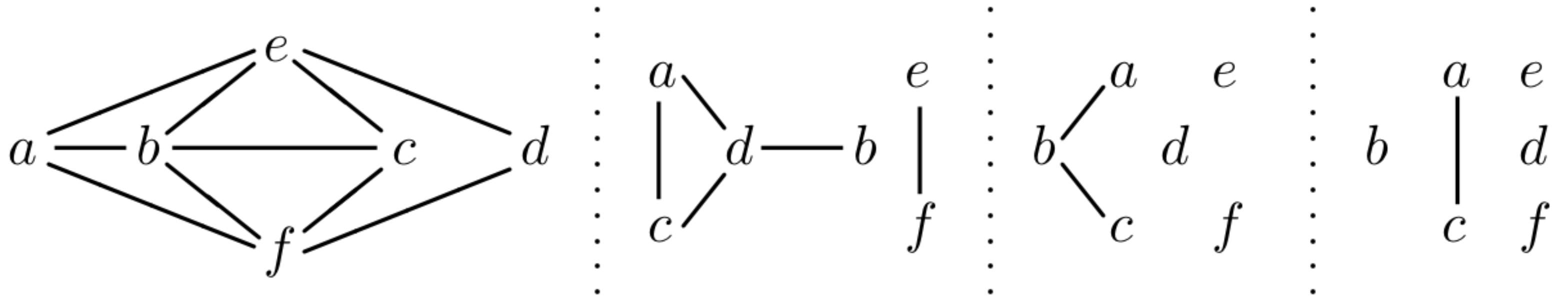

Equivalently, if it is P_4-free (every induced path has ≤2 edges).

D. Corneil, H. Lerchs & L. Stewart Burlingham,
"Complement reducible graphs" *Discrete Applied Mathematics* (1981)

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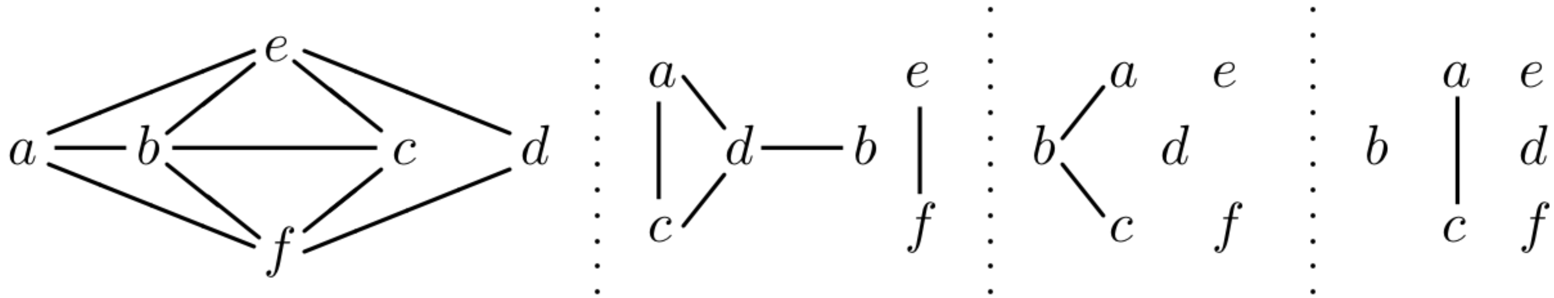

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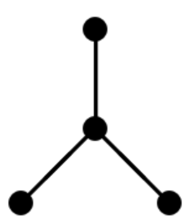
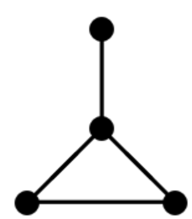
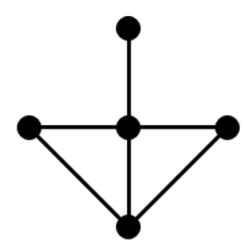


claw



paw



dart

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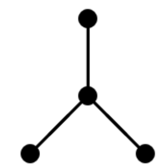
iff G is an intersection graph of open semicircular arcs of a circle.

iff G reduces to $K_{t(2)}$ (with $t \geq 1$) by repeatedly deleting universal vertices, then contracting selected complete graphs into vertices.

F. Bonomo, G. Durán, L.N. Grippo & M.D. Safe
"On structural results about circular-arc and circle graphs"
Journal of Graph Theory (2009)

Theorem: If G is a nontrivial connected graph, then every nontrivial component of G^c is complete bipartite iff G is a claw-free cograph.



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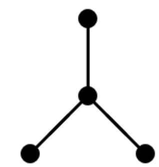

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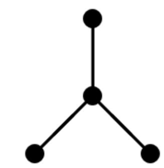
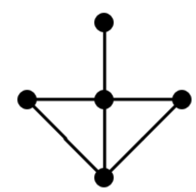


Theorem: If G is a nontrivial connected graph, then every nontrivial component of G^c is complete multipartite iff G is a dart-free cograph.



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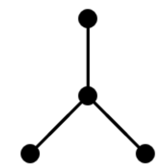

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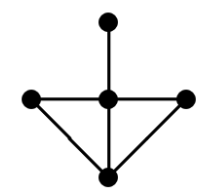

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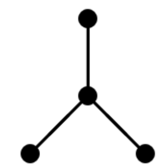

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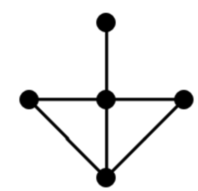
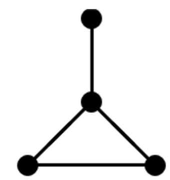


Theorem: If G is a nontrivial connected graph, then every nontrivial component of G^c is complete k-partite iff G is a $\{K_{1,k+1}$, dart$\}$-free cograph.

Lemma: A nontrivial connected graph is complete multipartite if and only if it is a paw-free cograph.

S. Olariu, "Paw-free graphs," Information Processing Letters (1988)



Lemma: A nontrivial connected graph is complete *k*-partite if and only if it is a $\{K_{k+1}, \text{paw}\}$-free cograph.

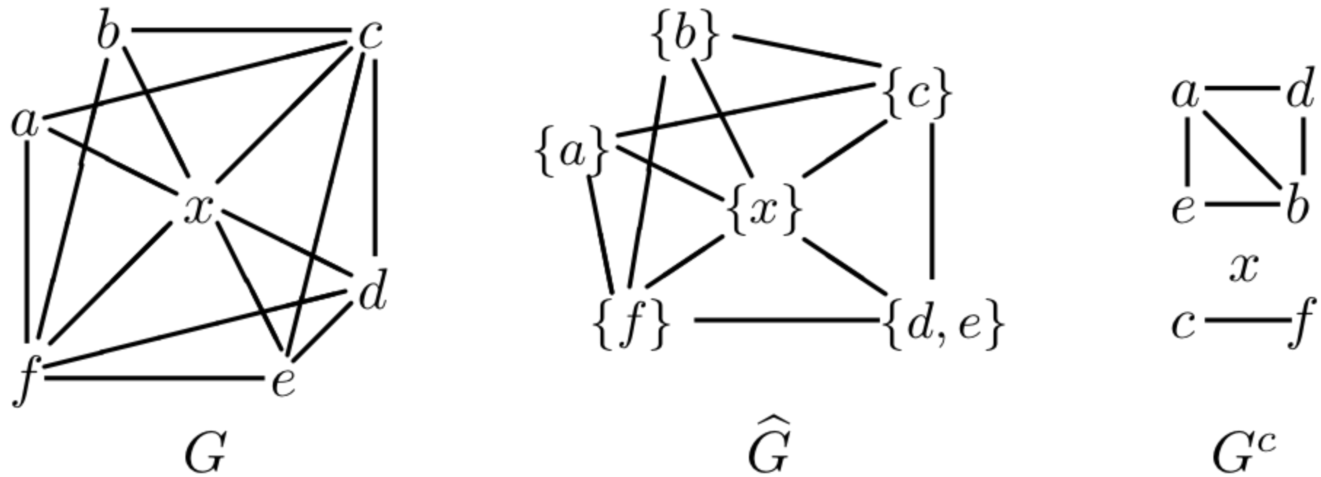
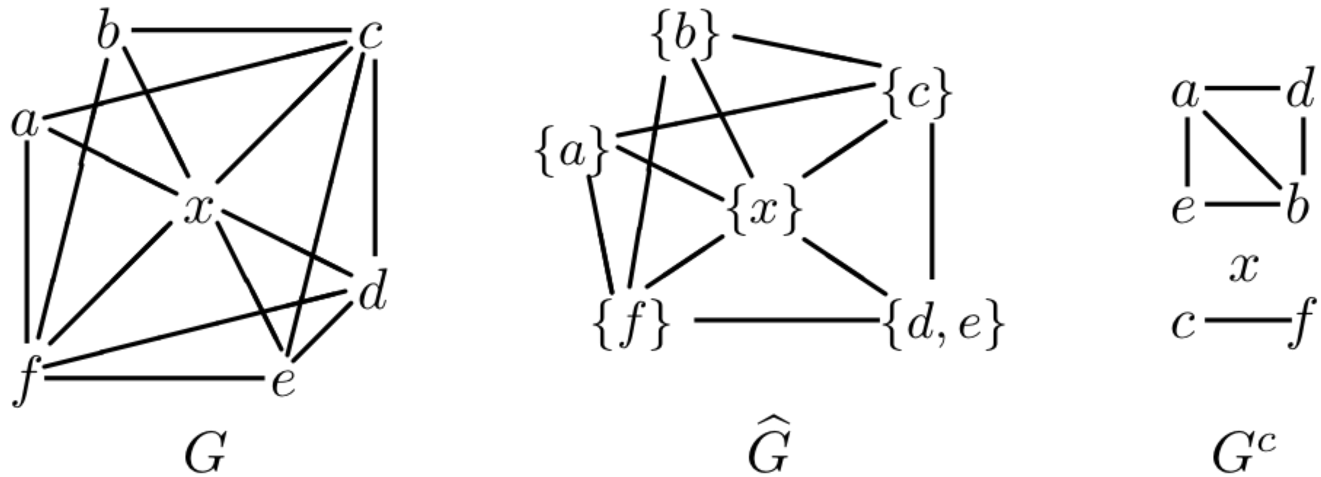


Figure 3: A dart-free (but not claw-free) cograph G.

L. Kuszner & M. Malafiejski,
"A polynomial algorithm for some
preemptive multiprocessor task scheduling problems,"
European Journal of Operations Research (2007)



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Lemma: A nontrivial connected graph *G* is a dart-free cograph if and only if $\widehat{G}$ is complete multipartite.

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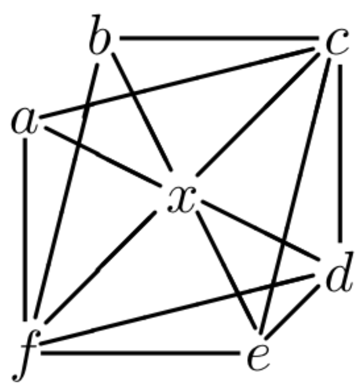
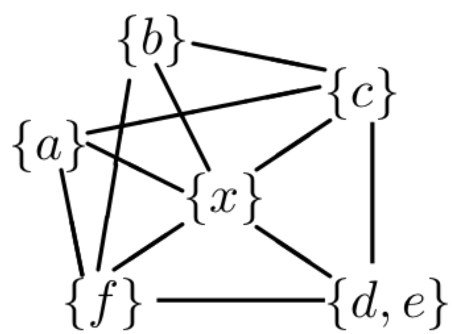
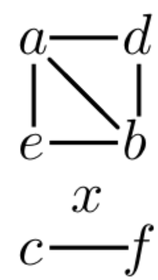


G



$\widehat{G}$

$\cong K_{1,2,3}$



G^c

$\cong K_{1,1,2} \cup K_{1,1} \cup K_1$

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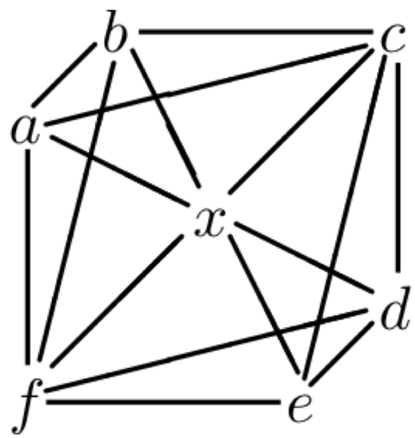
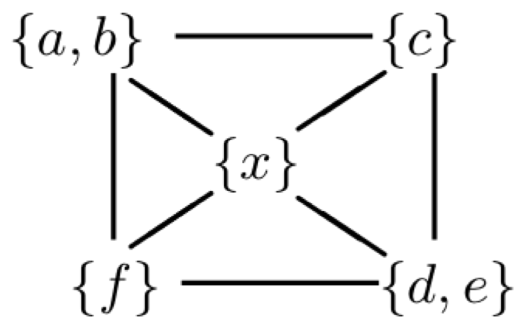


G



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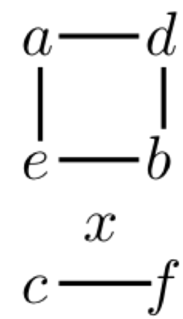

G^c

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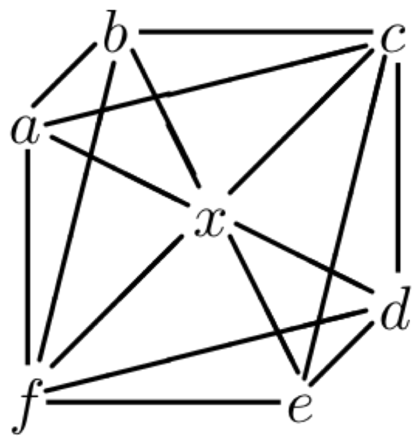
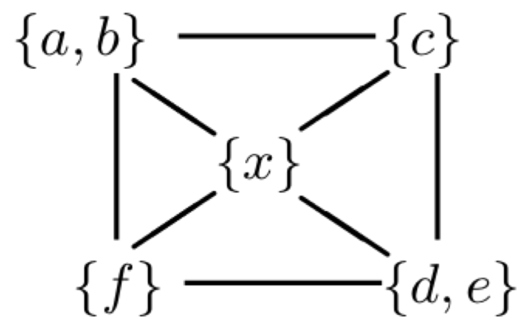


G



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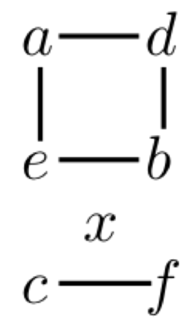

G^c

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Theorem: If G is a nontrivial connected graph, then G^c consists of k complete mutipartite (or trivial) components iff $\widehat{G}$ is complete k-partite.