Finding the Kth largest item in a list of n items

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This problem deals with finding the Kth largest element from an unordered list which consists of ’n’ elements.
Introduction

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- The easy approach of solving this problem is first sort the unordered list and then return the kth largest element.
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- The easy approach of solving this problem is first sort the unordered list and then return the kth largest element.

- This problem can be solved by two algorithms:
  - Selection algorithm.
  - Median of medians algorithm.
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This problem can be solved by two algorithms:
- Selection algorithm.
- Median of medians algorithm.

Median of medians algorithm is better than selection algorithm due to its worst case linear time performance.
The Median of Medians Algorithm was proposed by 5 great computer scientists they are Manuel Blum, Robert Floyd, Vaughan Pratt, Ron Rivest and Robert Tarjan in the year 1973.
Comparison

- In median of medians algorithm, we divide the list by 5 and then we sort the divided list, whereas in selection algorithm we directly sort the unordered list without dividing.

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Algorithm

1. Divide the list in to \( n/5 \) lists of 5 elements each.
2. Find the median in each sublist of 5 elements.
3. Recursively find the median of all the medians, call it \( m \).
4. Partition the list in to unique elements larger than \( 'm' \) (call this sublist \( L_1 \)) and those no longer than \( 'm' \) (call this sublists \( L_2 \)).
5. If \( K \leq |L_1| \), return selection \((L_1, K)\).
6. If \( K - 1 = |L_1| \), return \( 'm' \).
7. If \( K > |L_1| + 1 \), return selection\((L_2, K - |L_1| - 1)\).
Example

Find the 8th largest element i.e k = 8.

Unordered list
2 3 5 4 1 12 11 13
16 7 8 6 10 9 17 15
19 20 18 23 21 22
25 24 14

Dividing by 5
2 3 5 4 1
12 11 12 16 7
8 6 10 9 17
15 19 20 18 23
21 22 25 24 14
Example

Finding medians

\[
\begin{array}{cccc}
2 & 3 & 5 & 4 \\
12 & 11 & 13 & 16 \\
8 & 6 & 10 & 9 \\
15 & 19 & 20 & 18 \\
21 & 22 & 25 & 24 \\
\end{array}
\]

Median of medians

\[
\begin{array}{cccc}
5 & 13 & 10 & 20 \\
25 & \end{array}
\]
Example

$|L_2| = 9$

$|L_1| = 15$

12 11 13 16 17 15 19 20
18 23 21 22 25 24 14
Example

12 11 13 16 17
15 19 20 18 23
21 22 25 24 14

13 20 25

23 21 22 25 24

|L2| = 9

|L1| = 5

|L2| = 9

|L1| = 5
Example

\[12 \ 11 \ 13 \ 16 \ 17\]
\[15 \ 19 \ 18 \ 14\]

\[12 \ 11\]
\[13\]

\[14 \ 15 \ 16 \ 17 \ 18 \ 19\]

\(|L1| = 6; K = 2\]
Example

14 15

16

17 18 19

L1 = 3; k = 2

17

18

m

19

|L1| = 1; k = 2
Why 5?

- Dividing the list by 5 assures a worst-case split of 70 – 30.
- Atleast half of the medians are greater than the median-of-medians, hence atleast half of the \( n/5 \) blocks have atleast 3 elements and this gives a \( 3n/10 \) split, which means the other partition is \( 7n/10 \) in worst case.

That gives \( T(n) = T(n/5) + T(7n/10) + O(n) \).
Performance

- The best total running time of finding $K$th largest item in a list of $N$ items is $O(n\log n)$.

- Where running time of sorting $N$ items is $O(n\log n)$ and running time of returning the $K$th largest item is $O(1)$.

- The worst case running time is $O(n)$. 
Applications

- **Order Statistics**: Selection include for finding the smallest elements, largest elements and median elements.

- **Computer chess program**: Identifying the most promising candidates in computer chess program.

- **Salary Distribution**: Selection is used in salary distribution.

- **Filtering Outlying** elements such as noisy data.
References

- Donald Knuth. *The Art of Computer Programming*.