# Chromatic Numbers

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# Outline

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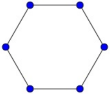
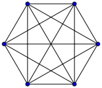
# Introduction

- **Definition** : The smallest number of colors necessary to color the nodes of graph so that no two adjacent nodes have the same color.



Chromatic Number of $C_6$ is : 2



Chromatic Number of $k_6$ is : 6

- In general, a graph with chromatic number $k$ is said to be an $k$-chromatic graph, and a graph with chromatic number$\leq k$ is said to be $k$-colorable.

# Inroduction contd.

- Chromatic number of a graph must be greater than or equal to its clique number.
- Determining the chromatic number of a general graph $G$ is well-known to be NP-hard.

| graph $G$ | $\chi(G)$ |
|---|---|
| complete graph $K_n$ | $n$ |
| cycle graph $C_n, n > 1$ | $\begin{cases} 3 & \text{for } n \text{ odd} \\ 2 & \text{for } n \text{ even} \end{cases}$ |
| star graph $S_n, n > 1$ | 2 |
| wheel graph $W_n, n > 2$ | $\begin{cases} 3 & \text{for } n \text{ odd} \\ 4 & \text{for } n \text{ even} \end{cases}$ |

# History

- Francis Guthrie postulated the four color conjecture while trying to color a map of the countries of England.
- Four Color Problem seems to have been mentioned for the first time in writing in an 1852 letter from A. De Morgan to W.R. Hamilton. Nobody thought at that time that it was the beginning of a new theory.
- The first proof was given by Kempe in 1879.It stood for more than 10 years until Heawood in 1890 found a mistake.
- The chromatic number problem is one of Karp's 21 NP-complete problems from 1972,and at approximately the same time various exponential-time algorithms weredeveloped based on backtracking and on the deletion-contraction recurrence of Zykov (1949).

# Statement of Problem

To determine the chromatic number of graph $G$, with vertices $v_1, v_2, \ldots, v_n$, where $n$ is the order of the graph.

# Dsatur Algorithm

## Algorithm

- Order the vertices $v_1, v_2, \ldots, v_n$ such that $d(v_1) \geq d(v_2) \geq \cdots \geq d(v_n)$.
- Assign color 1 to $v_1$, define $C_1 = \{v_1\}$, $r = 2 =$ the index of the next vertex to be colored, $j = 1 =$ the number of colors used up to now, $U =$ the set of current uncolored vertices $= V - \{v_1\}$.
- Determine $Satdeg(v)$ for $v \in U$. Define

$$PV = \{\acute{v} : Satdeg(\acute{v}) = max\{Satdeg(v) : v \in U\}\}$$

Choose the next vertex $\acute{v}$ to be colored if $d(\acute{v}) = max\{d(v) : v \in PV\}$

# Dsatur contd.

## Algorithm

- Let

$$\acute{i} = \begin{cases} \infty, & \text{if } Satdeg(\acute{v}) = j \\ \min\{i{:}x_{i\acute{v}} = 0, 1 \leq i \leq j\}, & \text{if otherwise} \end{cases}$$

  and $i*$ =min$\{\acute{i}, j+1\}$. *Color the vertex with color* i∗, add $\acute{v}$ to $C_{i*}$ and update $r$, $j$ and $U$.
- Repeat steps (c) and (d) until all the vertices are colored.

# Applications

## General

- schedules (conferences and events)
- programs (school programs)
- timetable (trains)
- distribution of items:
  - animals which can and cannot live together (distribution of species: fishes, spiders, snakes)
  - plants that can and cannot to be kept together
  - food which can and cannot be consumed together
  - people who can and cannot stay together (celebrations, wedding table)
  - determination of radio frequencies so that they don't detect each other
- suduko

# Applications Contd.

## Related to Computers

- Register Allocation
- Pattern Matching

# References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